
We adopt the following conventions in the integral tables:

1. A constant of integration must be included with all indefinite integrals.
2. All angles are measured in radians; inverse trigonometric and hyperbolic functions represent principal values.
3. Logarithmic expressions are to base $e = 2.71828 \dots$, unless otherwise specified, and are to be evaluated for the absolute value of the arguments involved therein.
4. The natural logarithm function is denoted as $\log x$.
5. The variables n and m usually denote integers. The denominator of the expressions shown is not allowed to be zero; this may require that $a \neq 0$ or $m \neq n$ or some other similar statement.
6. When inverse trigonometric functions occur in the integrals, be sure that any replacements made for them are strictly in accordance with the rules for such functions. This causes little difficulty when the argument of the inverse trigonometric function is positive, because all angles involved are in the first quadrant. However, if the argument is negative, special care must be used. Thus, if $u > 0$ then

$$\sin^{-1} u = \cos^{-1} \sqrt{1 - u^2} = \csc^{-1} \frac{1}{u} = \dots$$

However, if $u < 0$, then

$$\sin^{-1} u = -\cos^{-1} \sqrt{1 - u^2} = -\pi + \csc^{-1} \frac{1}{u} = \dots$$

5.4 TABLE OF INDEFINITE INTEGRALS

5.4.1 ELEMENTARY FORMS

1. $\int a \, dx = ax.$
2. $\int a f(x) \, dx = a \int f(x) \, dx.$
3. $\int \phi(y(x)) \, dx = \int \frac{\phi(y)}{y'} \, dy$, where $y' = \frac{dy}{dx}.$
4. $\int (u + v) \, dx = \int u \, dx + \int v \, dx$, where u and v are any functions of $x.$
5. $\int u \, dv = u \int dv - \int v \, du = uv - \int v \, du.$
6. $\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx.$

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7. $\int x^n dx = \frac{x^{n+1}}{n+1}$, except when $n = -1$.
 8. $\int \frac{dx}{x} = \log x$.
 9. $\int \frac{f'(x)}{f(x)} dx = \log f(x)$, $(df(x) = f'(x) dx)$.
 10. $\int \frac{f'(x)}{2\sqrt{f(x)}} dx = \sqrt{f(x)}$, $(df(x) = f'(x) dx)$.
 11. $\int e^x dx = e^x$.
 12. $\int e^{ax} dx = \frac{e^{ax}}{a}$.
 13. $\int b^{ax} dx = \frac{b^{ax}}{a \log b}$, $b > 0$.
 14. $\int \log x dx = x \log x - x$.
 15. $\int a^x dx = \frac{a^x}{\log a}$, $a > 0$.
 16. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$.
 17. $\int \frac{dx}{a^2 - x^2} = \begin{cases} \frac{1}{a} \tanh^{-1} \frac{x}{a}, \\ \text{or} \\ \frac{1}{2a} \log \frac{a+x}{a-x}, \end{cases} \quad a^2 > x^2.$
 18. $\int \frac{dx}{x^2 - a^2} = \begin{cases} -\frac{1}{a} \coth^{-1} \frac{x}{a}, \\ \text{or} \\ \frac{1}{2a} \log \frac{x-a}{x+a}, \end{cases} \quad x^2 > a^2.$
 19. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \begin{cases} \sin^{-1} \frac{x}{|a|}, \\ \text{or} \\ -\cos^{-1} \frac{x}{|a|}, \end{cases} \quad a^2 > x^2.$
 20. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log \left(x + \sqrt{x^2 \pm a^2} \right).$
 21. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{|a|} \sec^{-1} \frac{x}{a}.$
 22. $\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a} \log \left(\frac{a + \sqrt{a^2 \pm x^2}}{x} \right).$
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5.4.2 FORMS CONTAINING $a + bx$

23. $\int (a + bx)^n dx = \frac{(a + bx)^{n+1}}{(n+1)b}$, $n \neq -1$.
24. $\int x(a + bx)^n dx = \frac{1}{b^2(n+2)}(a + bx)^{n+2} - \frac{a}{b^2(n+1)}(a + bx)^{n+1},$
 $n \neq -1, n \neq -2.$

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25. $\int x^2(a+bx)^n dx = \frac{1}{b^3} \left[\frac{(a+bx)^{n+3}}{n+3} - 2a \frac{(a+bx)^{n+2}}{n+2} + a^2 \frac{(a+bx)^{n+1}}{n+1} \right],$
 $n \neq -1, \quad n \neq -2, \quad n \neq -3.$
26. $\int x^m(a+bx)^n dx =$

$$\begin{cases} \frac{x^{m+1}(a+bx)^n}{m+n+1} + \frac{an}{m+n+1} \int x^m(a+bx)^{n-1} dx, \\ \text{or} \\ \frac{1}{a(n+1)} \left[-x^{m+1}(a+bx)^{n+1} + (m+n+2) \int x^m(a+bx)^{n+1} dx \right], \\ \text{or} \\ \frac{1}{b(m+n+1)} \left[x^m(a+bx)^{n+1} - ma \int x^{m-1}(a+bx)^n dx \right]. \end{cases}$$
27. $\int \frac{dx}{a+bx} = \frac{1}{b} \log |a+bx|.$
28. $\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}.$
29. $\int \frac{dx}{(a+bx)^3} = -\frac{1}{2b(a+bx)^2}.$
30. $\int \frac{x}{a+bx} dx = \begin{cases} \frac{1}{b^2} [a+bx - a \log(a+bx)], \\ \text{or} \\ \frac{x}{b} - \frac{a}{b^2} \log(a+bx). \end{cases}$
31. $\int \frac{x}{(a+bx)^2} dx = \frac{1}{b^2} \left[\log(a+bx) + \frac{a}{a+bx} \right].$
32. $\int \frac{x}{(a+bx)^n} dx = \frac{1}{b^2} \left[\frac{-1}{(n-2)(a+bx)^{n-2}} + \frac{a}{(n-1)(a+bx)^{n-1}} \right],$
 $n \neq 1, \quad n \neq 2.$
33. $\int \frac{x^2}{a+bx} dx = \frac{1}{b^3} \left(\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2 \log(a+bx) \right).$
34. $\int \frac{x^2}{(a+bx)^2} dx = \frac{1}{b^3} \left(a+bx - 2a \log(a+bx) - \frac{a^2}{a+bx} \right).$
35. $\int \frac{x^2}{(a+bx)^3} dx = \frac{1}{b^3} \left(\log(a+bx) + \frac{2a}{a+bx} - \frac{a^2}{2(a+bx)^2} \right).$
36. $\int \frac{x^2}{(a+bx)^n} dx = \frac{1}{b^3} \left[\frac{-1}{(n-3)(a+bx)^{n-3}} + \frac{2a}{(n-2)(a+bx)^{n-2}} \right.$
 $\left. - \frac{a^2}{(n-1)(a+bx)^{n-1}} \right], \quad n \neq 1, \quad n \neq 2, \quad n \neq 3.$
37. $\int \frac{dx}{x(a+bx)} = -\frac{1}{a} \log \frac{a+bx}{x}.$
38. $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2} \log \frac{a+bx}{x}.$
39. $\int \frac{dx}{x(a+bx)^3} = \frac{1}{a^3} \left[\frac{1}{2} \left(\frac{2a+bx}{a+bx} \right)^2 - \log \frac{a+bx}{x} \right].$
40. $\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2} \log \frac{a+bx}{x}.$

$$41. \int \frac{dx}{x^3(a+bx)} = \frac{2bx-a}{2a^2x^2} + \frac{b^2}{a^3} \log \frac{x}{a+bx}.$$

$$42. \int \frac{dx}{x^2(a+bx)^2} = -\frac{a+2bx}{a^2x(a+bx)} + \frac{2b}{a^3} \log \frac{a+bx}{x}.$$

5.4.3 FORMS CONTAINING $c^2 \pm x^2$ AND $x^2 - c^2$

$$43. \int \frac{dx}{c^2 + x^2} = \frac{1}{c} \tan^{-1} \frac{x}{c}.$$

$$44. \int \frac{dx}{c^2 - x^2} = \frac{1}{2c} \log \frac{c+x}{c-x}, \quad c^2 > x^2.$$

$$45. \int \frac{dx}{x^2 - c^2} = \frac{1}{2c} \log \frac{x-c}{x+c}, \quad x^2 > c^2.$$

$$46. \int \frac{x}{c^2 \pm x^2} dx = \pm \frac{1}{2} \log (c^2 \pm x^2).$$

$$47. \int \frac{x}{(c^2 \pm x^2)^{n+1}} dx = \mp \frac{1}{2n(c^2 \pm x^2)^n}, \quad n \neq 0.$$

$$48. \int \frac{dx}{(c^2 \pm x^2)^n} = \frac{1}{2c^2(n-1)} \left[\frac{x}{(c^2 \pm x^2)^{n-1}} + (2n-3) \int \frac{dx}{(c^2 \pm x^2)^{n-1}} \right].$$

$$49. \int \frac{dx}{(x^2 - c^2)^n} = \frac{1}{2c^2(n-1)} \left[-\frac{x}{(x^2 - c^2)^{n-1}} - (2n-3) \int \frac{dx}{(x^2 - c^2)^{n-1}} \right].$$

$$50. \int \frac{x}{x^2 - c^2} dx = \frac{1}{2} \log (x^2 - c^2).$$

$$51. \int \frac{x}{(x^2 - c^2)^{n+1}} dx = -\frac{1}{2n(x^2 - c^2)^n}.$$

5.4.4 FORMS CONTAINING $a + bx$ AND $c + dx$

$$u = a + bx, \quad v = c + dx, \quad \text{and } k = ad - bc. \quad (\text{If } k = 0, \text{ then } v = (c/a)u.)$$

$$52. \int \frac{dx}{uv} = \frac{1}{k} \log \left(\frac{v}{u} \right).$$

$$53. \int \frac{x}{uv} dx = \frac{1}{k} \left(\frac{a}{b} \log u - \frac{c}{d} \log v \right).$$

$$54. \int \frac{dx}{u^2v} = \frac{1}{k} \left(\frac{1}{u} + \frac{d}{k} \log \frac{v}{u} \right).$$

$$55. \int \frac{x}{u^2v} dx = -\frac{a}{bku} - \frac{c}{k^2} \log \frac{v}{u}.$$

$$56. \int \frac{x^2}{u^2v} dx = \frac{a^2}{b^2ku} + \frac{1}{k^2} \left(\frac{c^2}{d} \log v + \frac{a(k-bc)}{b^2} \log u \right).$$

$$57. \int \frac{dx}{u^n v^m} = \frac{1}{k(m-1)} \left[\frac{-1}{u^{n-1}v^{m-1}} - b(m+n-2) \int \frac{dx}{u^n v^{m-1}} \right].$$

$$58. \int \frac{u}{v} dx = \frac{bx}{d} + \frac{k}{d^2} \log v.$$

$$59. \int \frac{u^m}{v^n} dx = \begin{cases} -\frac{1}{k(n-1)} \left[\frac{u^{m+1}}{v^{n-1}} + b(n-m-2) \int \frac{u^m}{v^{n-1}} dx \right], \\ \text{or} \\ -\frac{1}{d(n-m-1)} \left[\frac{u^m}{v^{n-1}} + mk \int \frac{u^{m-1}}{v^n} dx \right], \\ \text{or} \\ -\frac{1}{d(n-1)} \left[\frac{u^m}{v^{n-1}} - mb \int \frac{u^{m-1}}{v^{n-1}} dx \right]. \end{cases}$$

5.4.5 FORMS CONTAINING $a + bx^n$

$$60. \int \frac{dx}{a + bx^2} = \frac{1}{\sqrt{ab}} \tan^{-1} \frac{x\sqrt{ab}}{a}, \quad ab > 0.$$

$$61. \int \frac{dx}{a + bx^2} = \begin{cases} \frac{1}{2\sqrt{-ab}} \log \frac{a + x\sqrt{-ab}}{a - x\sqrt{-ab}}, & ab < 0, \\ \text{or} \\ \frac{1}{\sqrt{-ab}} \tanh^{-1} \frac{x\sqrt{-ab}}{a}, & ab < 0. \end{cases}$$

$$62. \int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a}.$$

$$63. \int \frac{x}{a + bx^2} dx = \frac{1}{2b} \log(a + bx^2).$$

$$64. \int \frac{x^2}{a + bx^2} dx = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a + bx^2}.$$

$$65. \int \frac{dx}{(a + bx^2)^2} = \frac{x}{2a(a + bx^2)} + \frac{1}{2a} \int \frac{dx}{a + bx^2}.$$

$$66. \int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \log \frac{a + bx}{a - bx}.$$

$$67. \int \frac{dx}{(a + bx^2)^{m+1}} = \begin{cases} \frac{1}{2ma} \frac{x}{(a + bx^2)^m} + \frac{2m-1}{2ma} \int \frac{dx}{(a + bx^2)^m}, \\ \text{or} \\ \frac{(2m)!}{(m!)^2} \left[\frac{x}{2a} \sum_{r=1}^m \frac{r!(r-1)!}{(4a)^{m-r}(2r)!(a + bx^2)^r} + \frac{1}{(4a)^m} \int \frac{dx}{a + bx^2} \right]. \end{cases}$$

$$68. \int \frac{x dx}{(a + bx^2)^{m+1}} = -\frac{1}{2bm(a + bx^2)^m}, \quad m \neq 0.$$

$$69. \int \frac{x^2 dx}{(a + bx^2)^{m+1}} = -\frac{x}{2mb(a + bx^2)^m} + \frac{1}{2mb} \int \frac{dx}{(a + bx^2)^m}, \quad m \neq 0.$$

$$70. \int \frac{dx}{x(a + bx^2)} = \frac{1}{2a} \log \frac{x^2}{a + bx^2}.$$

$$71. \int \frac{dx}{x^2(a + bx^2)} = -\frac{1}{ax} - \frac{b}{a} \int \frac{dx}{a + bx^2}.$$

$$72. \int \frac{dx}{x(a + bx^2)^{m+1}} = \begin{cases} \frac{1}{2am(a + bx^2)^m} + \frac{1}{a} \int \frac{dx}{x(a + bx^2)^m}, \\ \text{or} \\ \frac{1}{2a^{m+1}} \left[\sum_{r=1}^m \frac{a^r}{r(a + bx^2)^r} + \log \frac{x^2}{a + bx^2} \right]. \end{cases}$$

$$\begin{aligned}
73. \int \frac{dx}{x^2(a+bx^2)^{m+1}} &= \frac{1}{a} \int \frac{dx}{x^2(a+bx^2)^m} - \frac{b}{a} \int \frac{dx}{(a+bx^2)^{m+1}}. \\
74. \int \frac{dx}{a+bx^3} &= \frac{k}{3a} \left[\frac{1}{2} \log \frac{(k+x)^3}{a+bx^3} + \sqrt{3} \tan^{-1} \frac{2x-k}{k\sqrt{3}} \right], \quad k = \sqrt[3]{\frac{a}{b}}. \\
75. \int \frac{x dx}{a+bx^3} &= \frac{1}{3bk} \left[\frac{1}{2} \log \frac{a+bx^3}{(k+x)^3} + \sqrt{3} \tan^{-1} \frac{2x-k}{k\sqrt{3}} \right], \quad k = \sqrt[3]{\frac{a}{b}}. \\
76. \int \frac{x^2 dx}{a+bx^3} &= \frac{1}{3b} \log a+bx^3. \\
77. \int \frac{dx}{a+bx^4} &= \begin{cases} \frac{k}{2a} \left[\frac{1}{2} \log \frac{x^2+2kx+2k^2}{x^2-2kx+2k^2} + \tan^{-1} \frac{2kx}{2k^2-x^2} \right], & ab > 0, \quad k = \left(\frac{a}{4b}\right)^{1/4}, \\ \text{or} \\ \frac{k}{2a} \left[\frac{1}{2} \log \frac{x+k}{x-k} + \tan^{-1} \frac{x}{k} \right], & ab < 0, \quad k = \left(-\frac{a}{b}\right)^{1/4}. \end{cases} \\
78. \int \frac{x}{a+bx^4} dx &= \frac{1}{2bk} \tan^{-1} \frac{x^2}{k}, \quad ab > 0, \quad k = \sqrt{\frac{a}{b}}. \\
79. \int \frac{x}{a+bx^4} dx &= \frac{1}{4bk} \log \frac{x^2-k}{x^2+k}, \quad ab < 0, \quad k = \sqrt{-\frac{a}{b}}. \\
80. \int \frac{x^2}{a+bx^4} dx &= \frac{1}{4bk} \left[\frac{1}{2} \log \frac{x^2-2kx+2k^2}{x^2+2kx+2k^2} + \tan^{-1} \frac{2kx}{2k^2-x^2} \right], \\
&\quad ab > 0, \quad k = \left(\frac{a}{4b}\right)^{1/4}. \\
81. \int \frac{x^2 dx}{a+bx^4} &= \frac{1}{4bk} \left[\log \frac{x-k}{x+k} + 2 \tan^{-1} \frac{x}{k} \right], \quad ab < 0, \quad k = \sqrt[4]{-\frac{a}{b}}. \\
82. \int \frac{x^3 dx}{a+bx^4} &= \frac{1}{4b} \log(a+bx^4). \\
83. \int \frac{dx}{x(a+bx^n)} &= \frac{1}{an} \log \frac{x^n}{a+bx^n}, \quad n \neq 0. \\
84. \int \frac{dx}{(a+bx^n)^{m+1}} &= \frac{1}{a} \int \frac{dx}{(a+bx^n)^m} - \frac{b}{a} \int \frac{x^n dx}{(a+bx^n)^{m+1}}. \\
85. \int \frac{x^m dx}{(a+bx^n)^{p+1}} &= \frac{1}{b} \int \frac{x^{m-n} dx}{(a+bx^n)^p} - \frac{a}{b} \int \frac{x^{m-n} dx}{(a+bx^n)^{p+1}}. \\
86. \int \frac{dx}{x^m(a+bx^n)^{p+1}} &= \frac{1}{a} \int \frac{dx}{x^m(a+bx^n)^p} - \frac{b}{a} \int \frac{dx}{x^{m-n}(a+bx^n)^{p+1}}. \\
87. \int x^m(a+bx^n)^p dx &= \begin{cases} \frac{1}{b(np+m+1)} \left[x^{m-n+1}(a+bx^n)^{p+1} - a(m-n+1) \int x^{m-n}(a+bx^n)^p dx \right], \\ \text{or} \\ \frac{1}{np+m+1} \left[x^{m+1}(a+bx^n)^p + anp \int x^m(a+bx^n)^{p-1} dx \right], \\ \text{or} \\ \frac{1}{a(m+1)} \left[x^{m+1}(a+bx^n)^{p+1} - b(m+1+np+n) \int x^{m+n}(a+bx^n)^p dx \right], \\ \text{or} \\ \frac{1}{an(p+1)} \left[-x^{m+1}(a+bx^n)^{p+1} + (m+1+np+n) \int x^m(a+bx^n)^{p+1} dx \right]. \end{cases}
\end{aligned}$$

5.4.6 FORMS CONTAINING $c^3 \pm x^3$

88. $\int \frac{dx}{c^3 \pm x^3} = \pm \frac{1}{6c^2} \log \left(\frac{(c \pm x)^3}{c^3 \pm x^3} \right) + \frac{1}{c^2 \sqrt{3}} \tan^{-1} \frac{2x \mp c}{c\sqrt{3}}.$
89. $\int \frac{dx}{(c^3 \pm x^3)^2} = \frac{x}{3c^3(c^3 \pm x^3)} + \frac{2}{3c^3} \int \frac{dx}{c^3 \pm x^3}.$
90. $\int \frac{dx}{(c^3 \pm x^3)^{n+1}} = \frac{1}{3nc^3} \left[\frac{x}{(c^3 \pm x^3)^n} + (3n-1) \int \frac{dx}{(c^3 \pm x^3)^n} \right], \quad n \neq 0.$
91. $\int \frac{x dx}{c^3 \pm x^3} = \frac{1}{6c} \log \frac{c^3 \pm x^3}{(c \pm x)^3} \pm \frac{1}{c\sqrt{3}} \tan^{-1} \frac{2x \mp c}{c\sqrt{3}}.$
92. $\int \frac{x dx}{(c^3 \pm x^3)^2} = \frac{x^2}{3c^3(c^3 \pm x^3)} + \frac{1}{3c^3} \int \frac{x dx}{c^3 \pm x^3}.$
93. $\int \frac{x dx}{(c^3 \pm x^3)^{n+1}} = \frac{1}{3nc^3} \left[\frac{x^2}{(c^3 \pm x^3)^n} + (3n-2) \int \frac{x dx}{(c^3 \pm x^3)^n} \right], \quad n \neq 0.$
94. $\int \frac{x^2 dx}{c^3 \pm x^3} = \pm \frac{1}{3} \log (c^3 \pm x^3).$
95. $\int \frac{x^2 dx}{(c^3 \pm x^3)^{n+1}} = \mp \frac{1}{3n(c^3 \pm x^3)^n}, \quad n \neq 0.$
96. $\int \frac{dx}{x(c^3 \pm x^3)} = \frac{1}{3c^3} \log \frac{x^3}{c^3 \pm x^3}.$
97. $\int \frac{dx}{x(c^3 \pm x^3)^2} = \frac{1}{3c^3(c^3 \pm x^3)} + \frac{1}{3c^6} \log \frac{x^3}{c^3 \pm x^3}.$
98. $\int \frac{dx}{x(c^3 \pm x^3)^{n+1}} = \frac{1}{3nc^3(c^3 \pm x^3)^n} + \frac{1}{c^3} \int \frac{dx}{x(c^3 \pm x^3)^n}, \quad n \neq 0.$
99. $\int \frac{dx}{x^2(c^3 \pm x^3)} = -\frac{1}{c^3 x} \mp \frac{1}{c^3} \int \frac{x dx}{(c^3 \pm x^3)}.$
100. $\int \frac{dx}{x^2(c^3 \pm x^3)^{n+1}} = \frac{1}{c^3} \int \frac{dx}{x^2(c^3 \pm x^3)^n} \mp \frac{1}{c^3} \int \frac{x dx}{(c^3 \pm x^3)^{n+1}}.$

5.4.7 FORMS CONTAINING $c^4 \pm x^4$

101. $\int \frac{dx}{c^4 + x^4} = \frac{1}{2c^3 \sqrt{2}} \left[\frac{1}{2} \log \left(\frac{x^2 + cx\sqrt{2} + c^2}{x^2 - cx\sqrt{2} + c^2} \right) + \tan^{-1} \frac{cx\sqrt{2}}{c^2 - x^2} \right].$
102. $\int \frac{dx}{c^4 - x^4} = \frac{1}{2c^3} \left[\frac{1}{2} \log \frac{c+x}{c-x} + \tan^{-1} \frac{x}{c} \right].$
103. $\int \frac{x dx}{c^4 + x^4} = \frac{1}{2c^2} \tan^{-1} \frac{x^2}{c^2}.$
104. $\int \frac{x dx}{c^4 - x^4} = \frac{1}{4c^2} \log \frac{c^2 + x^2}{c^2 - x^2}.$
105. $\int \frac{x^2 dx}{c^4 + x^4} = \frac{1}{2c\sqrt{2}} \left[\frac{1}{2} \log \left(\frac{x^2 - cx\sqrt{2} + c^2}{x^2 + cx\sqrt{2} + c^2} \right) + \tan^{-1} \frac{cx\sqrt{2}}{c^2 - x^2} \right].$
106. $\int \frac{x^2 dx}{c^4 - x^4} = \frac{1}{2c} \left[\frac{1}{2} \log \frac{c+x}{c-x} - \tan^{-1} \frac{x}{c} \right].$

$$107. \int \frac{x^3 dx}{c^4 \pm x^4} = \pm \frac{1}{4} \log(c^4 \pm x^4).$$

5.4.8 FORMS CONTAINING $a + bx + cx^2$

$$X = a + bx + cx^2 \quad \text{and} \quad q = 4ac - b^2.$$

If $q = 0$, then $X = c \left(x + \frac{b}{2c}\right)^2$ and other formulae should be used.

$$108. \int \frac{dx}{X} = \begin{cases} \frac{2}{\sqrt{q}} \tan^{-1} \frac{2cx + b}{\sqrt{q}}, & q > 0, \\ \text{or} \\ \frac{-2}{\sqrt{-q}} \tanh^{-1} \frac{2cx + b}{\sqrt{-q}}, & q < 0, \\ \text{or} \\ \frac{1}{\sqrt{-q}} \log \frac{2cx + b - \sqrt{-q}}{2cx + b + \sqrt{-q}}, & q < 0. \end{cases}$$

$$109. \int \frac{dx}{X^2} = \frac{2cx + b}{qX} + \frac{2c}{q} \int \frac{dx}{X}.$$

$$110. \int \frac{dx}{X^3} = \frac{2cx + b}{q} \left(\frac{1}{2X^2} + \frac{3c}{qX} \right) + \frac{6c^2}{q^2} \int \frac{dx}{X}.$$

$$111. \int \frac{dx}{X^{n+1}} = \begin{cases} \frac{2cx + b}{nqX^n} + \frac{2(2n-1)c}{qn} \int \frac{dx}{X^n}, \\ \text{or} \\ \frac{(2n)!}{(n!)^2} \left(\frac{c}{q} \right)^n \left[\frac{2cx + b}{q} \sum_{r=1}^n \left(\frac{q}{cX} \right)^r \left(\frac{(r-1)!r!}{(2r)!} \right) + \int \frac{dx}{X} \right]. \end{cases}$$

$$112. \int \frac{x dx}{X} = \frac{1}{2c} \log X - \frac{b}{2c} \int \frac{dx}{X}.$$

$$113. \int \frac{x dx}{X^2} = -\frac{bx + 2a}{qX} - \frac{b}{q} \int \frac{dx}{X}.$$

$$114. \int \frac{x dx}{X^{n+1}} = -\frac{2a + bx}{nqX^n} - \frac{b(2n-1)}{nq} \int \frac{dx}{X^n}, \quad n \neq 0.$$

$$115. \int \frac{x^2 dx}{X} = \frac{x}{c} - \frac{b}{2c^2} \log X + \frac{b^2 - 2ac}{2c^2} \int \frac{dx}{X}.$$

$$116. \int \frac{x^2 dx}{X^2} = \frac{(b^2 - 2ac)x + ab}{cqX} + \frac{2a}{q} \int \frac{dx}{X}.$$

$$117. \int \frac{x^m dx}{X^{n+1}} = -\frac{x^{m-1}}{(2n-m+1)cX^n} - \frac{n-m+1}{2n-m+1} \frac{b}{c} \int \frac{x^{m-1}}{X^{n+1}} dx \\ + \frac{m-1}{2n-m+1} \frac{a}{c} \int \frac{x^{m-2}}{X^{n+1}} dx.$$

$$118. \int \frac{dx}{xX} = \frac{1}{2a} \log \frac{x^2}{X} - \frac{b}{2a} \int \frac{dx}{X}.$$

$$119. \int \frac{dx}{x^2 X} = \frac{b}{2a^2} \log \frac{X}{x^2} - \frac{1}{ax} + \left(\frac{b^2}{2a^2} - \frac{c}{a} \right) \int \frac{dx}{X}.$$

$$120. \int \frac{dx}{xX^n} = \frac{1}{2a(n-1)X^{n-1}} - \frac{b}{2a} \int \frac{dx}{X^n} + \frac{1}{a} \int \frac{dx}{xX^{n-1}}, \quad n \neq 1.$$

$$121. \int \frac{dx}{x^m X^{n+1}} = -\frac{1}{(m-1)ax^{m-1}X^n} - \frac{n+m-1}{m-1} \frac{b}{a} \int \frac{dx}{x^{m-1}X^{n+1}} - \frac{2n+m-1}{m-1} \frac{c}{a} \int \frac{dx}{x^{m-2}X^{n+1}}.$$

5.4.9 FORMS CONTAINING $\sqrt{a+bx}$

$$122. \int \sqrt{a+bx} dx = \frac{2}{3b} \sqrt{(a+bx)^3}.$$

$$123. \int x \sqrt{a+bx} dx = -\frac{2(2a-3bx)}{15b^2} \sqrt{(a+bx)^3}.$$

$$124. \int x^2 \sqrt{a+bx} dx = \frac{2(8a^2 - 12abx + 15b^2x^2)}{105b^3} \sqrt{(a+bx)^3}.$$

$$125. \int x^m \sqrt{a+bx} dx = \begin{cases} \frac{2}{b(2m+3)} \left[x^m \sqrt{(a+bx)^3} - ma \int x^{m-1} \sqrt{a+bx} dx \right], \\ \text{or} \\ \frac{2}{b^{m+1}} \sqrt{a+bx} \sum_{r=0}^m \frac{m!(-a)^{m-r}}{r!(m-r)!(2r+3)} (a+bx)^{r+1}. \end{cases}$$

$$126. \int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} + a \int \frac{dx}{x\sqrt{a+bx}}.$$

$$127. \int \frac{\sqrt{a+bx}}{x^m} dx = -\frac{1}{(m-1)a} \left[\frac{\sqrt{(a+bx)^3}}{x^{m-1}} + \frac{(2m-5)b}{2} \int \frac{\sqrt{a+bx}}{x^{m-1}} dx \right].$$

$$128. \int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b}.$$

$$129. \int \frac{x dx}{\sqrt{a+bx}} = -\frac{2(2a-bx)}{3b^2} \sqrt{a+bx}.$$

$$130. \int \frac{x^2 dx}{\sqrt{a+bx}} = \frac{2(8a^2 - 4abx + 3b^2x^2)}{15b^3} \sqrt{a+bx}.$$

$$131. \int \frac{x^m dx}{\sqrt{a+bx}} = \begin{cases} \frac{2}{(2m+1)b} \left[x^m \sqrt{a+bx} - ma \int \frac{x^{m-1}}{\sqrt{a+bx}} dx \right], \\ \text{or} \\ \frac{2(-a)^m \sqrt{a+bx}}{b^{m+1}} \sum_{r=0}^m \frac{(-1)^r m!(a+bx)^r}{(2r+1)r!(m-r)!a^r}. \end{cases}$$

$$132. \int \frac{dx}{x\sqrt{a+bx}} = \begin{cases} \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bx}{-a}}, & a < 0, \\ \text{or} \\ \frac{1}{\sqrt{a}} \log \left(\frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right), & a > 0. \end{cases}$$

$$133. \int \frac{dx}{x^2 \sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{ax} - \frac{b}{2a} \int \frac{dx}{x\sqrt{a+bx}}.$$

$$\begin{aligned}
134. \int \frac{dx}{x^n \sqrt{a+bx}} &= \begin{cases} -\frac{\sqrt{a+bx}}{(n-1)ax^{n-1}} - \frac{(2n-3)b}{(2n-2)a} \int \frac{dx}{x^{n-1} \sqrt{a+bx}}, \\ \text{or} \\ \frac{(2n-2)!}{[(n-1)!]^2} \left[-\frac{\sqrt{a+bx}}{a} \sum_{r=1}^{n-1} \frac{r!(r-1)!}{x^r (2r)!} \left(-\frac{b}{4a}\right)^{n-r-1} + \left(-\frac{b}{4a}\right)^{n-1} \int \frac{dx}{x \sqrt{a+bx}} \right]. \end{cases} \\
135. \int (a+bx)^{\pm n/2} dx &= \frac{2(a+bx)^{(2\pm n)/2}}{b(2\pm n)}. \\
136. \int x(a+bx)^{\pm n/2} dx &= \frac{2}{b^2} \left[\frac{(a+bx)^{(4\pm n)/2}}{4\pm n} - \frac{a(a+bx)^{(2\pm n)/2}}{2\pm n} \right]. \\
137. \int \frac{dx}{x(a+bx)^{n/2}} &= \frac{1}{a} \int \frac{dx}{x(a+bx)^{(n-2)/2}} - \frac{b}{a} \int \frac{dx}{(a+bx)^{n/2}}. \\
138. \int \frac{(a+bx)^{n/2}}{x} dx &= b \int (a+bx)^{(n-2)/2} dx + a \int \frac{(a+bx)^{(n-2)/2}}{x} dx.
\end{aligned}$$

5.4.10 FORMS CONTAINING $\sqrt{a+bx}$ AND $\sqrt{c+dx}$

and $\sqrt{c+dx}$

$$u = a + bx, \quad v = c + dx, \quad k = ad - bc.$$

If $k = 0$, then $v = \frac{c}{a}u$, and other formulae should be used.

$$139. \int \frac{dx}{\sqrt{uv}} = \begin{cases} \frac{2}{\sqrt{bd}} \tanh^{-1} \frac{\sqrt{bd}uv}{bv}, & bd > 0, k < 0, \\ \frac{2}{\sqrt{bd}} \tanh^{-1} \frac{\sqrt{bd}uv}{du}, & bd > 0, k > 0, \\ \frac{1}{\sqrt{bd}} \log \frac{(bv + \sqrt{bd}uv)^2}{v}, & bd > 0, \\ \frac{2}{\sqrt{-bd}} \tan^{-1} \frac{\sqrt{-bd}uv}{bv}, & bd < 0, \\ -\frac{1}{\sqrt{-bd}} \sin^{-1} \left(\frac{2bdx + ad + bc}{|k|} \right), & bd < 0. \end{cases}$$

$$140. \int \sqrt{uv} dx = \frac{k + 2bv}{4bd} \sqrt{uv} - \frac{k^2}{8bd} \int \frac{dx}{\sqrt{uv}}.$$

$$141. \int \frac{dx}{v\sqrt{u}} = \begin{cases} \frac{1}{\sqrt{kd}} \log \frac{d\sqrt{u} - \sqrt{kd}}{d\sqrt{u} + \sqrt{kd}}, & kd > 0, \\ \text{or} \\ \frac{1}{\sqrt{kd}} \log \frac{(d\sqrt{u} - \sqrt{kd})^2}{v}, & kd > 0, \\ \text{or} \\ \frac{2}{\sqrt{-kd}} \tan^{-1} \frac{d\sqrt{u}}{\sqrt{-kd}}, & kd < 0. \end{cases}$$

$$142. \int \frac{x dx}{\sqrt{uv}} = \frac{\sqrt{uv}}{bd} - \frac{ad + bc}{2bd} \int \frac{dx}{\sqrt{uv}}.$$

$$\begin{aligned}
143. \int \frac{dx}{v\sqrt{uv}} &= -\frac{2\sqrt{uv}}{kv}. \\
144. \int \frac{v dx}{\sqrt{uv}} &= \frac{\sqrt{uv}}{b} - \frac{k}{2b} \int \frac{dx}{\sqrt{uv}}. \\
145. \int \sqrt{\frac{v}{u}} dx &= \frac{v}{|v|} \int \frac{v dx}{\sqrt{uv}}. \\
146. \int v^m \sqrt{u} dx &= \frac{1}{(2m+3)d} \left(2v^{m+1} \sqrt{u} + k \int \frac{v^m dx}{\sqrt{u}} \right). \\
147. \int \frac{dx}{v^m \sqrt{u}} &= -\frac{1}{(m-1)k} \left(\frac{\sqrt{u}}{v^{m-1}} + \left(m - \frac{3}{2} \right) b \int \frac{dx}{v^{m-1} \sqrt{u}} \right), \quad m \neq 1. \\
148. \int \frac{v^m}{\sqrt{u}} dx &= \begin{cases} \frac{2}{b(2m+1)} \left(v^m \sqrt{u} - mk \int \frac{v^{m-1}}{\sqrt{u}} dx \right), \\ \text{or} \\ \frac{2(m!)^2 \sqrt{u}}{b(2m+1)!} \sum_{r=0}^m \left(-\frac{4k}{b} \right)^{m-r} \frac{(2r)!}{(r!)^2} v^r. \end{cases}
\end{aligned}$$

5.4.11 FORMS CONTAINING $\sqrt{x^2 \pm a^2}$

$$\begin{aligned}
149. \int \sqrt{x^2 \pm a^2} dx &= \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \log \left(x + \sqrt{x^2 \pm a^2} \right) \right]. \\
150. \int \frac{dx}{\sqrt{x^2 \pm a^2}} &= \log \left(x + \sqrt{x^2 \pm a^2} \right). \\
151. \int \frac{dx}{x \sqrt{x^2 - a^2}} &= \frac{1}{|a|} \sec^{-1} \frac{x}{a}. \\
152. \int \frac{dx}{x \sqrt{x^2 + a^2}} &= -\frac{1}{a} \log \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right). \\
153. \int \frac{\sqrt{x^2 + a^2}}{x} dx &= \sqrt{x^2 + a^2} - a \log \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right). \\
154. \int \frac{\sqrt{x^2 - a^2}}{x} dx &= \sqrt{x^2 - a^2} - |a| \sec^{-1} \frac{x}{a}. \\
155. \int \frac{x}{\sqrt{x^2 \pm a^2}} dx &= \sqrt{x^2 \pm a^2}. \\
156. \int x \sqrt{x^2 \pm a^2} dx &= \frac{1}{3} \sqrt{(x^2 \pm a^2)^3}. \\
157. \int \sqrt{(x^2 \pm a^2)^3} dx &= \frac{1}{4} \left[x \sqrt{(x^2 \pm a^2)^3} \pm \frac{3a^2 x}{2} \sqrt{x^2 \pm a^2} \right. \\
&\quad \left. + \frac{3a^4}{2} \log \left(x + \sqrt{x^2 \pm a^2} \right) \right]. \\
158. \int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} &= \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}}. \\
159. \int \frac{x}{\sqrt{(x^2 \pm a^2)^3}} dx &= \frac{-1}{\sqrt{x^2 \pm a^2}}. \\
160. \int x \sqrt{(x^2 \pm a^2)^3} dx &= \frac{1}{5} \sqrt{(x^2 \pm a^2)^5}. \\
161. \int x^2 \sqrt{x^2 \pm a^2} dx &= \frac{x}{4} \sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8} x \sqrt{x^2 \pm a^2} - \frac{a^4}{8} \log \left(x + \sqrt{x^2 \pm a^2} \right).
\end{aligned}$$

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162. $\int x^3 \sqrt{x^2 + a^2} dx = \frac{1}{15} (3x^2 - 2a^2) \sqrt{(x^2 + a^2)^3}.$
163. $\int x^3 \sqrt{x^2 - a^2} dx = \frac{1}{5} \sqrt{(x^2 - a^2)^5} + \frac{a^2}{3} \sqrt{(x^2 - a^2)^3}.$
164. $\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \log \left(x + \sqrt{x^2 \pm a^2} \right).$
165. $\int \frac{x^3}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{3} \sqrt{(x^2 \pm a^2)^3} \mp a^2 \sqrt{x^2 \pm a^2}.$
166. $\int \frac{dx}{x^2 \sqrt{x^2 \pm a^2}} dx = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}.$
167. $\int \frac{dx}{x^3 \sqrt{x^2 + a^2}} dx = -\frac{\sqrt{x^2 + a^2}}{2a^2 x^2} + \frac{1}{2a^3} \log \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right).$
168. $\int \frac{dx}{x^3 \sqrt{x^2 - a^2}} dx = \frac{\sqrt{x^2 - a^2}}{2a^2 x^2} + \frac{1}{2|a|^3} \sec^{-1} \frac{x}{a}.$
169. $\int x^2 \sqrt{(x^2 \pm a^2)^3} dx = \frac{x}{6} \sqrt{(x^2 \pm a^2)^5} \mp \frac{a^2 x}{24} \sqrt{(x^2 \pm a^2)^3} - \frac{a^4 x}{16} \sqrt{x^2 \pm a^2}$
 $\mp \frac{a^6}{16} \log \left(x + \sqrt{x^2 \pm a^2} \right).$
170. $\int x^3 \sqrt{(x^2 \pm a^2)^3} dx = \frac{1}{7} \sqrt{(x^2 \pm a^2)^7} \mp \frac{a^2}{5} \sqrt{(x^2 \pm a^2)^5}.$
171. $\int \frac{\sqrt{x^2 \pm a^2}}{x^2} dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \log \left(x + \sqrt{x^2 \pm a^2} \right).$
172. $\int \frac{\sqrt{x^2 + a^2}}{x^3} dx = -\frac{\sqrt{x^2 + a^2}}{2x^2} - \frac{1}{2a} \log \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right).$
173. $\int \frac{\sqrt{x^2 - a^2}}{x^3} dx = -\frac{\sqrt{x^2 - a^2}}{2x^2} + \frac{1}{2|a|} \sec^{-1} \frac{x}{a}.$
174. $\int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \mp \frac{\sqrt{(x^2 \pm a^2)^3}}{3a^2 x^3}.$
175. $\int \frac{x^2 dx}{\sqrt{(x^2 \pm a^2)^3}} = -\frac{x}{\sqrt{x^2 \pm a^2}} + \log \left(x + \sqrt{x^2 \pm a^2} \right).$
176. $\int \frac{x^3 dx}{\sqrt{(x^2 \pm a^2)^3}} = \sqrt{x^2 \pm a^2} \pm \frac{a^2}{\sqrt{x^2 \pm a^2}}.$
177. $\int \frac{dx}{x \sqrt{(x^2 + a^2)^3}} = \frac{1}{a^2 \sqrt{x^2 + a^2}} - \frac{1}{a^3} \log \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right).$
178. $\int \frac{dx}{x \sqrt{(x^2 - a^2)^3}} = -\frac{1}{a^2 \sqrt{x^2 - a^2}} - \frac{1}{|a|^3} \sec^{-1} \frac{x}{a}.$
179. $\int \frac{dx}{x^2 \sqrt{(x^2 \pm a^2)^3}} = -\frac{1}{a^4} \left[\frac{\sqrt{x^2 \pm a^2}}{x} + \frac{x}{\sqrt{x^2 \pm a^2}} \right].$
180. $\int \frac{dx}{x^3 \sqrt{(x^2 + a^2)^3}} = -\frac{3 + a^2}{2a^4 \sqrt{x^2 + a^2}} + \frac{3}{2a^5} \log \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right).$
181. $\int \frac{dx}{x^3 \sqrt{(x^2 - a^2)^3}} = \frac{1}{2a^2 x^2 \sqrt{x^2 - a^2}} - \frac{3}{2a^4 \sqrt{x^2 - a^2}} - \frac{3}{2|a|^5} \sec^{-1} \frac{x}{a}.$
182. $\int \frac{x^m dx}{\sqrt{x^2 \pm a^2}} = \frac{1}{m} x^{m-1} \sqrt{x^2 \pm a^2} \mp \frac{m-1}{m} a^2 \int \frac{x^{m-2}}{\sqrt{x^2 \pm a^2}} dx.$

$$\begin{aligned}
183. \int \frac{x^{2m} dx}{\sqrt{x^2 \pm a^2}} &= \frac{(2m)!}{2^{2m}(m!)^2} \left[\sqrt{x^2 \pm a^2} \sum_{r=1}^m \frac{r!(r-1)!}{(2r)!} (\mp a^2)^{m-r} (2x)^{2r-1} \right. \\
&\quad \left. + (\mp a^2)^m \log \left(x + \sqrt{x^2 \pm a^2} \right) \right]. \\
184. \int \frac{x^{2m+1} dx}{\sqrt{x^2 \pm a^2}} &= \sqrt{x^2 \pm a^2} \sum_{r=0}^m \frac{(2r)!(m!)^2}{(2m+1)!(r!)^2} (\mp 4a^2)^{m-r} x^{2r}. \\
185. \int \frac{dx}{x^m \sqrt{x^2 \pm a^2}} &= \mp \frac{\sqrt{x^2 \pm a^2}}{(m-1)a^2 x^{m-1}} \mp \frac{(m-2)}{(m-1)a^2} \int \frac{dx}{x^{m-2} \sqrt{x^2 \pm a^2}}. \\
186. \int \frac{dx}{x^{2m} \sqrt{x^2 \pm a^2}} &= \sqrt{x^2 \pm a^2} \sum_{r=0}^{m-1} \frac{(m-1)!m!(2r)!2^{2m-2r-1}}{(r!)^2(2m)! (\mp a^2)^{m-r} x^{2r+1}}. \\
187. \int \frac{dx}{x^{2m+1} \sqrt{x^2 + a^2}} &= \frac{(2m)!}{(m!)^2} \left[\frac{\sqrt{x^2 + a^2}}{a^2} \sum_{r=1}^m (-1)^{m-r+1} \frac{r!(r-1)!}{2(2r)!(4a^2)^{m-r} x^{2r}} \right. \\
&\quad \left. + \frac{(-1)^{m+1}}{2^{2m} a^{2m+1}} \log \left(\frac{\sqrt{x^2 + a^2} + a}{x} \right) \right]. \\
188. \int \frac{dx}{x^{2m+1} \sqrt{x^2 - a^2}} &= \frac{(2m)!}{(m!)^2} \left[\frac{\sqrt{x^2 - a^2}}{a^2} \sum_{r=1}^m \frac{r!(r-1)!}{2(2r)!(4a^2)^{m-r} x^{2r}} \right. \\
&\quad \left. + \frac{1}{2^{2m} |a|^{2m+1}} \sec^{-1} \frac{x}{a} \right]. \\
189. \int \frac{dx}{(x-a)\sqrt{x^2 - a^2}} &= -\frac{\sqrt{x^2 - a^2}}{a(x-a)}. \\
190. \int \frac{dx}{(x+a)\sqrt{x^2 - a^2}} &= \frac{\sqrt{x^2 - a^2}}{a(x+a)}.
\end{aligned}$$

5.4.12 FORMS CONTAINING $\sqrt{a^2 - x^2}$

$$\begin{aligned}
191. \int \sqrt{a^2 - x^2} dx &= \frac{1}{2} \left(x \sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{|a|} \right). \\
192. \int \frac{dx}{\sqrt{a^2 - x^2}} &= \sin^{-1} \frac{x}{|a|} = -\cos^{-1} \frac{x}{|a|}. \\
193. \int \frac{dx}{x \sqrt{a^2 - x^2}} &= -\frac{1}{a} \log \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right). \\
194. \int \frac{\sqrt{a^2 - x^2}}{x} dx &= \sqrt{a^2 - x^2} - a \log \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right). \\
195. \int \frac{x}{\sqrt{a^2 - x^2}} dx &= -\sqrt{a^2 - x^2}. \\
196. \int x \sqrt{a^2 - x^2} dx &= -\frac{1}{3} \sqrt{(a^2 - x^2)^3}. \\
197. \int \sqrt{(a^2 - x^2)^3} dx &= \frac{1}{4} \left(x \sqrt{(a^2 - x^2)^3} + \frac{3a^2 x}{2} \sqrt{a^2 - x^2} + \frac{3a^4}{2} \sin^{-1} \frac{x}{|a|} \right). \\
198. \int \frac{dx}{\sqrt{(a^2 - x^2)^3}} &= \frac{x}{a^2 \sqrt{a^2 - x^2}}. \\
199. \int \frac{x}{\sqrt{(a^2 - x^2)^3}} dx &= \frac{1}{\sqrt{a^2 - x^2}}.
\end{aligned}$$

200. $\int x \sqrt{(a^2 - x^2)^3} dx = -\frac{1}{5} \sqrt{(a^2 - x^2)^5}.$

201. $\int x^2 \sqrt{a^2 - x^2} dx = -\frac{x}{4} \sqrt{(a^2 - x^2)^3} + \frac{a^2}{8} \left(x \sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{|a|} \right).$

202. $\int x^3 \sqrt{a^2 - x^2} dx = \left(-\frac{1}{5} x^2 - \frac{2}{15} a^2 \right) \sqrt{(a^2 - x^2)^3}.$

203. $\int x^2 \sqrt{(a^2 - x^2)^3} dx = -\frac{1}{6} x \sqrt{(a^2 - x^2)^5} + \frac{a^2 x}{24} \sqrt{(a^2 - x^2)^3}$
 $+ \frac{a^4 x}{16} \sqrt{a^2 - x^2} + \frac{a^6}{16} \sin^{-1} \frac{x}{|a|}.$

204. $\int x^3 \sqrt{(a^2 - x^2)^3} dx = \frac{1}{7} \sqrt{(a^2 - x^2)^7} - \frac{a^2}{5} \sqrt{(a^2 - x^2)^5}.$

205. $\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{|a|}.$

206. $\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x}.$

207. $\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1} \frac{x}{|a|}.$

208. $\int \frac{\sqrt{a^2 - x^2}}{x^3} dx = -\frac{\sqrt{a^2 - x^2}}{2x^2} + \frac{1}{2a} \log \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right).$

209. $\int \frac{\sqrt{a^2 - x^2}}{x^4} dx = -\frac{\sqrt{(a^2 - x^2)^3}}{3a^2 x^3}.$

210. $\int \frac{x^2 dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{\sqrt{a^2 - x^2}} - \sin^{-1} \frac{x}{|a|}.$

211. $\int \frac{x^3 dx}{\sqrt{a^2 - x^2}} = -\frac{2}{3} \sqrt{(a^2 - x^2)^3} - x^2 \sqrt{a^2 - x^2}.$

212. $\int \frac{x^3 dx}{\sqrt{(a^2 - x^2)^3}} = 2\sqrt{a^2 - x^2} + \frac{x^2}{\sqrt{a^2 - x^2}} = \frac{a^2}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2}.$

213. $\int \frac{dx}{x^3 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{2a^2 x^2} - \frac{1}{2a^3} \log \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right).$

214. $\int \frac{dx}{x \sqrt{(a^2 - x^2)^3}} = \frac{1}{a^2 \sqrt{a^2 - x^2}} - \frac{1}{a^3} \log \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right).$

215. $\int \frac{dx}{x^2 \sqrt{(a^2 - x^2)^3}} = \frac{1}{a^4} \left(-\frac{\sqrt{a^2 - x^2}}{x} + \frac{x}{\sqrt{a^2 - x^2}} \right).$

216. $\int \frac{dx}{x^3 \sqrt{(a^2 - x^2)^3}} = \frac{3 - a^2}{2a^4 \sqrt{a^2 - x^2}} - \frac{3}{2a^5} \log \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right).$

217. $\int \frac{x^m}{\sqrt{a^2 - x^2}} dx = -\frac{x^{m-1} \sqrt{a^2 - x^2}}{m} + \frac{(m-1)a^2}{m} \int \frac{x^{m-2}}{\sqrt{a^2 - x^2}} dx.$

218. $\int \frac{x^{2m}}{\sqrt{a^2 - x^2}} dx = \frac{(2m)!}{(m!)^2} \left[-\sqrt{a^2 - x^2} \sum_{r=1}^m \frac{r!(r-1)!}{2^{2m-2r+1} (2r)!} a^{2m-2r} x^{2r-1} \right.$
 $\left. + \frac{a^{2m}}{2^{2m}} \sin^{-1} \frac{x}{|a|} \right].$

$$\begin{aligned}
219. \int \frac{x^{2m+1}}{\sqrt{a^2 - x^2}} dx &= -\sqrt{a^2 - x^2} \sum_{r=0}^m \frac{(2r)!(m!)^2}{(2m+1)!(r!)^2} (4a^2)^{m-r} x^{2r}. \\
220. \int \frac{dx}{x^m \sqrt{a^2 - x^2}} &= -\frac{\sqrt{a^2 - x^2}}{(m-1)a^2 x^{m-1}} + \frac{(m-2)}{(m-1)a^2} \int \frac{dx}{x^{m-2} \sqrt{a^2 - x^2}}. \\
221. \int \frac{dx}{x^{2m} \sqrt{a^2 - x^2}} &= -\sqrt{a^2 - x^2} \sum_{r=0}^{m-1} \frac{(m-1)!m!(2r)!2^{2m-2r-1}}{(r!)^2(2m)!a^{2m-2r} x^{2r+1}}. \\
222. \int \frac{dx}{x^{2m+1} \sqrt{a^2 - x^2}} &= \frac{(2m)!}{(m!)^2} \left[-\frac{\sqrt{a^2 - x^2}}{a^2} \sum_{r=1}^m \frac{r!(r-1)!}{2(2r)!(4a^2)^{m-r} x^{2r}} \right. \\
&\quad \left. + \frac{1}{2^{2m} a^{2m+1}} \log \left(\frac{a - \sqrt{a^2 - x^2}}{x} \right) \right]. \\
223. \int \frac{dx}{(b^2 - x^2)\sqrt{a^2 - x^2}} &= \begin{cases} \frac{1}{2b\sqrt{a^2 - b^2}} \log \left(\frac{(b\sqrt{a^2 - x^2} + x\sqrt{a^2 - b^2})^2}{b^2 - x^2} \right), & a^2 > b^2, \\ \text{or} \\ \frac{1}{b\sqrt{b^2 - a^2}} \tan^{-1} \frac{x\sqrt{b^2 - a^2}}{b\sqrt{a^2 - x^2}}, & b^2 > a^2. \end{cases} \\
224. \int \frac{dx}{(b^2 + x^2)\sqrt{a^2 - x^2}} &= \frac{1}{b\sqrt{a^2 + b^2}} \tan^{-1} \frac{x\sqrt{a^2 + b^2}}{b\sqrt{a^2 - x^2}}. \\
225. \int \frac{\sqrt{a^2 - x^2}}{b^2 + x^2} dx &= \frac{\sqrt{a^2 + b^2}}{|b|} \sin^{-1} \frac{x\sqrt{a^2 + b^2}}{|a|\sqrt{x^2 + b^2}} - \sin^{-1} \frac{x}{|a|}, \quad b^2 > a^2.
\end{aligned}$$

5.4.13 FORMS CONTAINING $\sqrt{a + bx + cx^2}$

$$X = a + bx + cx^2, \quad q = 4ac - b^2, \quad \text{and} \quad k = 4c/q.$$

$$\text{If } q = 0, \text{ then } \sqrt{X} = \sqrt{c} \left| x + \frac{b}{2c} \right|.$$

$$\begin{aligned}
226. \int \frac{dx}{\sqrt{X}} &= \begin{cases} \frac{1}{\sqrt{c}} \log \left(\frac{2\sqrt{cX} + 2cx + b}{\sqrt{q}} \right), & c > 0, \\ \text{or} \\ \frac{1}{\sqrt{c}} \sinh^{-1} \frac{2cx + b}{\sqrt{q}}, & c > 0, \\ \text{or} \\ -\frac{1}{\sqrt{-c}} \sin^{-1} \frac{2cx + b}{\sqrt{-q}}, & c < 0. \end{cases} \\
227. \int \frac{dx}{X\sqrt{X}} &= \frac{2(2cx + b)}{q\sqrt{X}}. \\
228. \int \frac{dx}{X^2\sqrt{X}} &= \frac{2(2cx + b)}{3q\sqrt{X}} \left(\frac{1}{X} + 2k \right). \\
229. \int \frac{dx}{X^n\sqrt{X}} &= \begin{cases} \frac{2(2cx + b)\sqrt{X}}{(2n-1)qX^n} + \frac{2k(n-1)}{2n-1} \int \frac{dx}{X^{n-1}\sqrt{X}}, \\ \text{or} \\ \frac{(2cx + b)(n!)(n-1)!4^n k^{n-1}}{q(2n)!\sqrt{X}} \sum_{r=0}^{n-1} \frac{(2r)!}{(4kX)^r (r!)^2}. \end{cases}
\end{aligned}$$

$$\begin{aligned}
230. \int \sqrt{X} dx &= \frac{(2cx + b)\sqrt{X}}{4c} + \frac{1}{2k} \int \frac{dx}{\sqrt{X}}. \\
231. \int X\sqrt{X} dx &= \frac{(2cx + b)\sqrt{X}}{8c} \left(X + \frac{3}{2k} \right) + \frac{3}{8k^2} \int \frac{dx}{\sqrt{X}}. \\
232. \int X^2\sqrt{X} dx &= \frac{(2cx + b)\sqrt{X}}{12c} \left(X^2 + \frac{5X}{4k} + \frac{15}{8k^2} \right) + \frac{5}{16k^3} \int \frac{dx}{\sqrt{X}}. \\
233. \int X^n\sqrt{X} dx &= \begin{cases} \frac{(2cx + b)X^n\sqrt{X}}{4(n+1)c} + \frac{2n+1}{2(n+1)k} \int X^{n-1}\sqrt{X} dx, \\ \text{or} \\ \frac{(2n+2)!}{[(n+1)!]^2 (4k)^{n+1}} \left[\frac{k(2cx + b)\sqrt{X}}{c} \sum_{r=0}^n \frac{r!(r+1)!(4kX)^r}{(2r+2)!} + \int \frac{dx}{\sqrt{X}} \right]. \end{cases} \\
234. \int \frac{x dx}{\sqrt{X}} &= \frac{\sqrt{X}}{c} - \frac{b}{2c} \int \frac{dx}{\sqrt{X}}. \\
235. \int \frac{x dx}{X\sqrt{X}} &= -\frac{2(bx + 2a)}{q\sqrt{X}}. \\
236. \int \frac{x dx}{X^n\sqrt{X}} &= -\frac{\sqrt{X}}{(2n-1)cX^n} - \frac{b}{2c} \int \frac{dx}{X^n\sqrt{X}}. \\
237. \int \frac{x^2 dx}{\sqrt{X}} &= \left(\frac{x}{2c} - \frac{3b}{4c^2} \right) \sqrt{X} + \frac{3b^2 - 4ac}{8c^2} \int \frac{dx}{\sqrt{X}}. \\
238. \int \frac{x^2 dx}{X\sqrt{X}} &= \frac{(2b^2 - 4ac)x + 2ab}{cq\sqrt{X}} + \frac{1}{c} \int \frac{dx}{\sqrt{X}}. \\
239. \int \frac{x^2 dx}{X^n\sqrt{X}} &= \frac{(2b^2 - 4ac)x + 2ab}{(2n-1)cqX^{n-1}\sqrt{X}} + \frac{4ac + (2n-3)b^2}{(2n-1)cq} \int \frac{dx}{X^{n-1}\sqrt{X}}. \\
240. \int \frac{x^3 dx}{\sqrt{X}} &= \left(\frac{x^2}{3c} - \frac{5bx}{12c^2} + \frac{5b^2}{8c^3} - \frac{2a}{3c^2} \right) \sqrt{X} + \left(\frac{3ab}{4c^2} - \frac{5b^3}{16c^3} \right) \int \frac{dx}{\sqrt{X}}. \\
241. \int \frac{x^n dx}{\sqrt{X}} &= \frac{1}{nc} x^{n-1} \sqrt{X} - \frac{(2n-1)b}{2nc} \int \frac{x^{n-1} dx}{\sqrt{X}} - \frac{(n-1)a}{nc} \int \frac{x^{n-2} dx}{\sqrt{X}}. \\
242. \int x\sqrt{X} dx &= \frac{X\sqrt{X}}{3c} - \frac{b(2cx + b)}{8c^2} \sqrt{X} - \frac{b}{4ck} \int \frac{dx}{\sqrt{X}}. \\
243. \int xX\sqrt{X} dx &= \frac{X^2\sqrt{X}}{5c} - \frac{b}{2c} \int X\sqrt{X} dx. \\
244. \int xX^n\sqrt{X} dx &= \frac{X^{n+1}\sqrt{X}}{(2n+3)c} - \frac{b}{2c} \int X^n\sqrt{X} dx. \\
245. \int x^2\sqrt{X} dx &= \left(x - \frac{5b}{6c} \right) \frac{X\sqrt{X}}{4c} + \frac{5b^2 - 4ac}{16c^2} \int \sqrt{X} dx. \\
246. \int \frac{dx}{x\sqrt{X}} &= \begin{cases} \frac{1}{\sqrt{-a}} \sin^{-1} \left(\frac{bx + 2a}{|x|\sqrt{-q}} \right), & a < 0, \\ \text{or} \\ -\frac{2\sqrt{X}}{bx}, & a = 0, \\ \text{or} \\ -\frac{1}{\sqrt{a}} \log \left(\frac{2\sqrt{aX} + bx + 2a}{x} \right), & a > 0. \end{cases}
\end{aligned}$$

$$\begin{aligned}
247. \int \frac{dx}{x^2 \sqrt{X}} &= -\frac{\sqrt{X}}{ax} - \frac{b}{2a} \int \frac{dx}{x \sqrt{X}}. \\
248. \int \frac{\sqrt{X}}{x} dx &= \sqrt{X} + \frac{b}{2} \int \frac{dx}{\sqrt{X}} + a \int \frac{dx}{x \sqrt{X}}. \\
249. \int \frac{\sqrt{X}}{x^2} dx &= -\frac{\sqrt{X}}{x} + \frac{b}{2} \int \frac{dx}{x \sqrt{X}} + c \int \frac{dx}{\sqrt{X}}.
\end{aligned}$$

5.4.14 FORMS CONTAINING $\sqrt{2ax - x^2}$

$$\begin{aligned}
250. \int \sqrt{2ax - x^2} dx &= \frac{1}{2} \left[(x - a) \sqrt{2ax - x^2} + a^2 \sin^{-1} \frac{x - a}{|a|} \right]. \\
251. \int \frac{dx}{\sqrt{2ax - x^2}} &= \begin{cases} \cos^{-1} \left(\frac{a - x}{|a|} \right), \\ \text{or} \\ \sin^{-1} \left(\frac{x - a}{|a|} \right). \end{cases} \\
252. \int x^n \sqrt{2ax - x^2} dx &= \begin{cases} -\frac{x^{n-1} \sqrt{(2ax - x^2)^3}}{n+2} + \frac{(2n+1)a}{n+2} \int x^{n-1} \sqrt{2ax - x^2} dx, \\ \text{or} \\ \sqrt{2ax - x^2} \left[\frac{x^{n+1}}{n+2} - \sum_{r=0}^n \frac{(2n+1)!(r!)^2 a^{n-r+1}}{2^{n-r} (2r+1)! (n+2)! n!} x^r \right] + \frac{(2n+1)! a^{n+2}}{2^n n! (n+2)!} \sin^{-1} \left(\frac{x - a}{|a|} \right). \end{cases} \\
253. \int \frac{\sqrt{2ax - x^2}}{x^n} dx &= \frac{\sqrt{(2ax - x^2)^3}}{(3-2n)ax^n} + \frac{n-3}{(2n-3)a} \int \frac{\sqrt{2ax - x^2}}{x^{n-1}} dx. \\
254. \int \frac{x^n dx}{\sqrt{2ax - x^2}} &= \begin{cases} -\frac{x^{n-1} \sqrt{2ax - x^2}}{n} + \frac{a(2n-1)}{n} \int \frac{x^{n-1}}{\sqrt{2ax - x^2}} dx, \\ \text{or} \\ -\sqrt{2ax - x^2} \sum_{r=1}^n \frac{(2n)! r! (r-1)! a^{n-r}}{2^{n-r} (2r)! (n!)^2} x^{r-1} + \frac{(2n)! a^n}{2^n (n!)^2} \sin^{-1} \left(\frac{x - a}{|a|} \right). \end{cases} \\
255. \int \frac{dx}{x^n \sqrt{2ax - x^2}} &= \begin{cases} \frac{\sqrt{2ax - x^2}}{a(1-2n)x^n} + \frac{n-1}{(2n-1)a} \int \frac{dx}{x^{n-1} \sqrt{2ax - x^2}}, \\ \text{or} \\ -\sqrt{2ax - x^2} \sum_{r=0}^{n-1} \frac{2^{n-r} (n-1)! n! (2r)!}{(2n)! (r!)^2 a^{n-r} x^{r+1}}. \end{cases} \\
256. \int \frac{dx}{\sqrt{(2ax - x^2)^3}} &= \frac{x - a}{a^2 \sqrt{2ax - x^2}}. \\
257. \int \frac{x dx}{\sqrt{(2ax - x^2)^3}} &= \frac{x}{a \sqrt{2ax - x^2}}.
\end{aligned}$$

5.4.15 MISCELLANEOUS ALGEBRAIC FORMS

$$258. \int \frac{dx}{\sqrt{2ax + x^2}} = \log \left(x + a + \sqrt{2ax + x^2} \right).$$

$$259. \int \sqrt{ax^2 + c} dx = \begin{cases} \frac{x}{2} \sqrt{ax^2 + c} + \frac{c}{2\sqrt{-a}} \sin^{-1} \left(x \sqrt{-\frac{a}{c}} \right), & a < 0, \\ \text{or} \\ \frac{x}{2} \sqrt{ax^2 + c} + \frac{c}{2\sqrt{a}} \log \left(x\sqrt{a} + \sqrt{ax^2 + c} \right), & a > 0. \end{cases}$$

$$260. \int \sqrt{\frac{1+x}{1-x}} dx = \sin^{-1} x - \sqrt{1-x^2}.$$

$$261. \int \frac{dx}{x\sqrt{ax^n + c}} = \begin{cases} \frac{1}{n\sqrt{c}} \log \frac{\sqrt{ax^n + c} - \sqrt{c}}{\sqrt{ax^n + c} + \sqrt{c}}, \\ \text{or} \\ \frac{2}{n\sqrt{c}} \log \frac{\sqrt{ax^n + c} - \sqrt{c}}{\sqrt{x^n}}, & c > 0, \\ \text{or} \\ \frac{2}{n\sqrt{-c}} \sec^{-1} \sqrt{-\frac{ax^n}{c}}, & c < 0. \end{cases}$$

$$262. \int \frac{dx}{\sqrt{ax^2 + c}} = \begin{cases} \frac{1}{\sqrt{-a}} \sin^{-1} \left(x \sqrt{-\frac{a}{c}} \right), & a < 0, \\ \text{or} \\ \frac{1}{\sqrt{a}} \log \left(x\sqrt{a} + \sqrt{ax^2 + c} \right), & a > 0. \end{cases}$$

$$263. \int (ax^2 + c)^{m+1/2} dx = \begin{cases} \frac{x(ax^2 + c)^{m+1/2}}{2(m+1)} + \frac{(2m+1)c}{2(m+1)} \int (ax^2 + c)^{m-1/2} dx, \\ \text{or} \\ x\sqrt{ax^2 + c} \sum_{r=0}^m \frac{(2m+1)!(r!)^2 c^{m-r}}{2^{2m-2r+1} m!(m+1)!(2r+1)!} (ax^2 + c)^r \\ + \frac{(2m+1)!c^{m+1}}{2^{2m+1} m!(m+1)!} \int \frac{dx}{\sqrt{ax^2 + c}}. \end{cases}$$

$$264. \int x(ax^2 + c)^{m+1/2} dx = \frac{(ax^2 + c)^{m+3/2}}{(2m+3)a}.$$

$$265. \int \frac{(ax^2 + c)^{m+1/2}}{x} dx = \begin{cases} \frac{(ax^2 + c)^{m+1/2}}{2m+1} + c \int \frac{(ax^2 + c)^{m-1/2}}{x} dx, \\ \text{or} \\ \sqrt{ax^2 + c} \sum_{r=0}^m \frac{c^{m-r} (ax^2 + c)^r}{2r+1} + c^{m+1} \int \frac{dx}{x\sqrt{ax^2 + c}}. \end{cases}$$

$$266. \int \frac{dx}{(ax^2 + c)^{m+1/2}} = \begin{cases} \frac{x}{(2m-1)c(ax^2 + c)^{m-1/2}} + \frac{2m-2}{(2m-1)c} \int \frac{dx}{(ax^2 + c)^{m-1/2}}, \\ \text{or} \\ \frac{x}{\sqrt{ax^2 + c}} \sum_{r=0}^{m-1} \frac{2^{2m-2r-1} (m-1)! m! (2r)!}{(2m)! (r!)^2 c^{m-r} (ax^2 + c)^r}. \end{cases}$$

$$267. \int \frac{dx}{x^m \sqrt{ax^2 + c}} = -\frac{\sqrt{ax^2 + c}}{(m-1)c x^{m-1}} - \frac{(m-2)a}{(m-1)c} \int \frac{dx}{x^{m-2} \sqrt{ax^2 + c}}, \quad m \neq 1.$$

$$\begin{aligned}
268. \int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx &= \frac{1}{\sqrt{2}} \log \left(\frac{x\sqrt{2} + \sqrt{1+x^4}}{1-x^2} \right). \\
269. \int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx &= \frac{1}{\sqrt{2}} \tan^{-1} \frac{x\sqrt{2}}{\sqrt{1+x^4}}. \\
270. \int \frac{dx}{x\sqrt{x^n+a^2}} &= -\frac{2}{na} \log \left(\frac{a + \sqrt{x^n+a^2}}{\sqrt{x^n}} \right). \\
271. \int \frac{dx}{x\sqrt{x^n-a^2}} &= -\frac{2}{na} \sin^{-1} \frac{a}{\sqrt{x^n}}. \\
272. \int \sqrt{\frac{x}{a^3-x^3}} dx &= \frac{2}{3} \sin^{-1} \left(\frac{x}{a} \right)^{3/2}.
\end{aligned}$$

5.4.16 FORMS INVOLVING TRIGONOMETRIC FUNCTIONS

$$\begin{aligned}
273. \int \sin ax \, dx &= -\frac{1}{a} \cos ax. \\
274. \int \cos ax \, dx &= \frac{1}{a} \sin ax. \\
275. \int \tan ax \, dx &= -\frac{1}{a} \log \cos ax = \frac{1}{a} \log \sec ax. \\
276. \int \cot ax \, dx &= \frac{1}{a} \log \sin ax = -\frac{1}{a} \log \csc ax. \\
277. \int \sec ax \, dx &= \frac{1}{a} \log (\sec ax + \tan ax) = \frac{1}{a} \log \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right). \\
278. \int \csc ax \, dx &= \frac{1}{a} \log (\csc ax - \cot ax) = \frac{1}{a} \log \tan \frac{ax}{2}. \\
279. \int \sin^2 ax \, dx &= \frac{x}{2} - \frac{1}{2a} \cos ax \sin ax = \frac{x}{2} - \frac{1}{4a} \sin 2ax. \\
280. \int \sin^3 ax \, dx &= -\frac{1}{3a} (\cos ax)(\sin^2 ax + 2). \\
281. \int \sin^4 ax \, dx &= \frac{3x}{8} - \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}. \\
282. \int \sin^n ax \, dx &= -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax \, dx. \\
283. \int \sin^{2m} ax \, dx &= -\frac{\cos ax}{a} \sum_{r=0}^{m-1} \frac{(2m)!(r!)^2}{2^{2m-2r}(2r+1)!(m!)^2} \sin^{2r+1} ax + \frac{(2m)!}{2^{2m}(m!)^2} x. \\
284. \int \sin^{2m+1} ax \, dx &= -\frac{\cos ax}{a} \sum_{r=0}^{m-1} \frac{2^{2m-2r}(m!)^2(2r)!}{(2m+1)!(r!)^2} \sin^{2r} ax. \\
285. \int \cos^2 ax \, dx &= \frac{1}{2}x + \frac{1}{2a} \sin ax \cos ax = \frac{1}{2}x + \frac{1}{4a} \sin 2ax. \\
286. \int \cos^3 ax \, dx &= \frac{1}{3a} \sin ax (\cos^2 ax + 2). \\
287. \int \cos^4 ax \, dx &= \frac{3}{8}x + \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}. \\
288. \int \cos^n ax \, dx &= \frac{1}{na} \cos^{n-1} ax \sin ax + \frac{n-1}{n} \int \cos^{n-2} ax \, dx.
\end{aligned}$$

$$\begin{aligned}
289. \int \cos^{2m} ax \, dx &= \frac{\sin ax}{a} \sum_{r=0}^{m-1} \frac{(2m)!(r!)^2}{2^{2m-2r}(2r+1)!(m!)^2} \cos^{2r+1} ax + \frac{(2m)!}{2^{2m}(m!)^2} x. \\
290. \int \cos^{2m+1} ax \, dx &= \frac{\sin ax}{a} \sum_{r=0}^m \frac{2^{2m-2r}(m!)^2(2r)!}{(2m+1)!(r!)^2} \cos^{2r} ax. \\
291. \int \frac{dx}{\sin^2 ax} &= \int \operatorname{cosec}^2 ax \, dx = -\frac{1}{a} \cot ax. \\
292. \int \frac{dx}{\sin^m ax} &= \int \operatorname{cosec}^m ax \, dx = -\frac{1}{a(m-1)} \frac{\cos ax}{\sin^{m-1} ax} + \frac{m-2}{m-1} \int \frac{dx}{\sin^{m-2} ax}. \\
293. \int \frac{dx}{\sin^{2m} ax} &= \int \operatorname{cosec}^{2m} ax \, dx = -\frac{1}{a} \cos ax \sum_{r=0}^{m-1} \frac{2^{2m-2r-1}(m-1)!m!(2r)!}{(2m)!(r!)^2 \sin^{2r+1} ax}. \\
294. \int \frac{dx}{\sin^{2m+1} ax} &= \int \operatorname{cosec}^{2m+1} ax \, dx = \\
&\quad -\frac{1}{a} \cos ax \sum_{r=0}^{m-1} \frac{(2m)!(r!)^2}{2^{2m-2r}(2r+1)!(m!)^2 \sin^{2r+2} ax} + \frac{1}{a} \frac{(2m)!}{2^{2m}(m!)^2} \log \tan \frac{ax}{2}. \\
295. \int \frac{dx}{\cos^2 ax} &= \int \sec^2 ax \, dx = \frac{1}{a} \tan ax. \\
296. \int \frac{dx}{\cos^m ax} &= \int \sec^m ax \, dx = \frac{1}{a(m-1)} \frac{\sin ax}{\cos^{m-1} ax} + \frac{m-2}{m-1} \int \frac{dx}{\cos^{m-2} ax}. \\
297. \int \frac{dx}{\cos^{2m} ax} &= \int \sec^{2m} ax \, dx = \frac{1}{a} \sin ax \sum_{r=0}^{m-1} \frac{2^{2m-2r-1}(m-1)!m!(2r)!}{(2m)!(r!)^2 \cos^{2r+1} ax}. \\
298. \int \frac{dx}{\cos^{2m+1} ax} &= \int \sec^{2m+1} ax \, dx = \frac{1}{a} \frac{(2m)!}{2^{2m}(m!)^2} \log (\sec ax + \tan ax) \\
&\quad + \frac{1}{a} \sin ax \sum_{r=0}^{m-1} \frac{(2m)!(r!)^2}{2^{2m-2r}(m!)^2(2r+1)! \cos^{2r+2} ax}. \\
299. \int (\sin mx)(\sin nx) \, dx &= \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)}, \quad m^2 \neq n^2. \\
300. \int (\cos mx)(\cos nx) \, dx &= \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)}, \quad m^2 \neq n^2. \\
301. \int (\sin ax)(\cos ax) \, dx &= \frac{1}{2a} \sin^2 ax. \\
302. \int (\sin mx)(\cos nx) \, dx &= -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)}, \quad m^2 \neq n^2. \\
303. \int (\sin^2 ax)(\cos^2 ax) \, dx &= -\frac{1}{32a} \sin 4ax + \frac{x}{8}. \\
304. \int (\sin ax)(\cos^m ax) \, dx &= -\frac{\cos^{m+1} ax}{(m+1)a}. \\
305. \int (\sin^m ax)(\cos ax) \, dx &= \frac{\sin^{m+1} ax}{(m+1)a}. \\
306. \int (\cos^m ax)(\sin^n ax) \, dx &= \\
&\quad \begin{cases} \frac{\cos^{m-1} ax \sin^{n+1} ax}{(m+n)a} + \frac{m-1}{m+n} \int (\cos^{m-2} ax)(\sin^n ax) \, dx, \\ \text{or} \\ -\frac{\cos^{m+1} ax \sin^{n-1} ax}{(m+n)a} + \frac{n-1}{m+n} \int (\cos^m ax)(\sin^{n-2} ax) \, dx. \end{cases}
\end{aligned}$$

$$\begin{aligned}
307. \int \frac{\cos^m ax}{\sin^n ax} dx &= \begin{cases} -\frac{\cos^{m+1} ax}{a(n-1)\sin^{n-1} ax} - \frac{m-n+2}{n-1} \int \frac{\cos^m ax}{\sin^{n-2} ax} dx, \\ \text{or} \\ \frac{\cos^{m-1} ax}{a(m-n)\sin^{n-1} ax} + \frac{m-1}{m-n} \int \frac{\cos^{m-2} ax}{\sin^n ax} dx. \end{cases} \\
308. \int \frac{\sin^m ax}{\cos^n ax} dx &= \begin{cases} \frac{\sin^{m+1} ax}{a(n-1)\cos^{n-1} ax} - \frac{m-n+2}{n-1} \int \frac{\sin^m ax}{\cos^{n-2} ax} dx, \\ \text{or} \\ -\frac{\sin^{m-1} ax}{a(m-n)\cos^{n-1} ax} + \frac{m-1}{m-n} \int \frac{\sin^{m-2} ax}{\cos^n ax} dx. \end{cases} \\
309. \int \frac{\sin ax}{\cos^2 ax} dx &= \frac{1}{a \cos ax} = \frac{\sec ax}{a}. \\
310. \int \frac{\sin^2 ax}{\cos ax} dx &= -\frac{1}{a} \sin ax + \frac{1}{a} \log \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right). \\
311. \int \frac{\cos ax}{\sin^2 ax} dx &= -\frac{\csc ax}{a} = -\frac{1}{a \sin ax}. \\
312. \int \frac{dx}{(\sin ax)(\cos ax)} &= \frac{1}{a} \log \tan ax. \\
313. \int \frac{dx}{(\sin ax)(\cos^2 ax)} &= \frac{1}{a} \left(\sec ax + \log \tan \frac{ax}{2} \right). \\
314. \int \frac{dx}{(\sin ax)(\cos^n ax)} &= \frac{1}{a(n-1)\cos^{n-1} ax} + \int \frac{dx}{(\sin ax)(\cos^{n-2} ax)}. \\
315. \int \frac{dx}{(\sin^2 ax)(\cos ax)} &= -\frac{1}{a} \csc ax + \frac{1}{a} \log \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right). \\
316. \int \frac{dx}{(\sin^2 ax)(\cos^2 ax)} &= -\frac{2}{a} \cot 2ax. \\
317. \int \frac{dx}{\sin^m ax \cos^n ax} &= \begin{cases} -\frac{1}{a(m-1)\sin^{m-1} ax \cos^{n-1} ax} + \frac{m+n-2}{m-1} \int \frac{dx}{\sin^{m-2} ax \cos^n ax}, \\ \text{or} \\ \frac{1}{a(n-1)\sin^{m-1} ax \cos^{n-1} ax} + \frac{m+n-2}{n-1} \int \frac{dx}{\sin^m ax \cos^{n-2} ax}. \end{cases} \\
318. \int \sin(a+bx) dx &= -\frac{1}{b} \cos(a+bx). \\
319. \int \cos(a+bx) dx &= \frac{1}{b} \sin(a+bx). \\
320. \int \frac{dx}{1 \pm \sin ax} &= \mp \frac{1}{a} \tan \left(\frac{\pi}{4} \mp \frac{ax}{2} \right). \\
321. \int \frac{dx}{1 + \cos ax} &= \frac{1}{a} \tan \frac{ax}{2}. \\
322. \int \frac{dx}{1 - \cos ax} &= -\frac{1}{a} \cot \frac{ax}{2}. \\
323. \int \frac{dx}{a + b \sin x} &= \begin{cases} \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left(\frac{a \tan \frac{x}{2} + b}{\sqrt{a^2 - b^2}} \right), \\ \text{or} \\ \frac{1}{\sqrt{b^2 - a^2}} \log \left(\frac{a \tan \frac{x}{2} + b - \sqrt{b^2 - a^2}}{a \tan \frac{x}{2} + b + \sqrt{b^2 - a^2}} \right). \end{cases}
\end{aligned}$$

$$\begin{aligned}
324. \int \frac{dx}{a + b \cos x} &= \begin{cases} \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \frac{\sqrt{a^2 - b^2} \tan \frac{x}{2}}{a + b}, \\ \text{or} \\ \frac{1}{\sqrt{b^2 - a^2}} \log \left(\frac{\sqrt{b^2 - a^2} \tan \frac{x}{2} + a + b}{\sqrt{b^2 - a^2} \tan \frac{x}{2} - a - b} \right). \end{cases} \\
325. \int \frac{dx}{a + b \sin x + c \cos x} &= \begin{cases} \frac{1}{\sqrt{b^2 + c^2 - a^2}} \log \left(\frac{b - \sqrt{b^2 + c^2 - a^2} + (a - c) \tan \frac{x}{2}}{b + \sqrt{b^2 + c^2 - a^2} + (a - c) \tan \frac{x}{2}} \right), & a \neq c, a^2 < b^2 + c^2, \\ \text{or} \\ \frac{2}{\sqrt{a^2 - b^2 - c^2}} \tan^{-1} \frac{b + (a - c) \tan \frac{x}{2}}{\sqrt{a^2 - b^2 - c^2}}, & a^2 > b^2 + c^2, \\ \text{or} \\ \frac{1}{a} \left[\frac{a - (b + c) \sin x - (b - c) \sin x}{a - (b + c) \sin x + (b - c) \sin x} \right], & a^2 = b^2 + c^2. \end{cases} \\
326. \int \frac{\sin^2 x}{a + b \cos^2 x} dx &= \frac{1}{b} \sqrt{\frac{a+b}{a}} \tan^{-1} \left(\sqrt{\frac{a}{a+b}} \tan x \right) - \frac{x}{b}, \quad ab > 0, |a| > |b|. \\
327. \int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} &= \frac{1}{ab} \tan^{-1} \left(\frac{b \tan x}{a} \right). \\
328. \int \frac{\cos^2 cx}{a^2 + b^2 \sin^2 cx} dx &= \frac{\sqrt{a^2 + b^2}}{ab^2 c} \tan^{-1} \frac{\sqrt{a^2 + b^2} \tan cx}{a} - \frac{x}{b^2}. \\
329. \int \frac{\sin cx \cos cx}{a \cos^2 cx + b \sin^2 cx} dx &= \frac{1}{2c(b-a)} \log (a \cos^2 cx + b \sin^2 cx), \quad a \neq b. \\
330. \int \frac{\cos cx}{a \cos cx + b \sin cx} dx &= \\
&\int \frac{dx}{a + b \tan cx} = \frac{1}{c(a^2 + b^2)} [acx + b \log (a \cos cx + b \sin cx)]. \\
331. \int \frac{\sin cx}{a \cos cx + b \sin cx} dx &= \\
&\int \frac{dx}{b + a \cot cx} = \frac{1}{c(a^2 + b^2)} [bcx - a \log (a \cos cx + b \sin cx)]. \\
332. \int \frac{dx}{a \cos^2 x + 2b \cos x \sin x + c \sin^2 x} &= \\
&\begin{cases} \frac{1}{2\sqrt{b^2 - ac}} \log \left(\frac{c \tan x + b - \sqrt{b^2 - ac}}{c \tan x + b + \sqrt{b^2 - ac}} \right), & b^2 > ac, \\ \text{or} \\ \frac{1}{\sqrt{ac - b^2}} \tan^{-1} \left(\frac{c \tan x + b}{\sqrt{ac - b^2}} \right), & b^2 < ac, \\ \text{or} \\ -\frac{1}{c \tan x + b}, & b^2 = ac. \end{cases} \\
333. \int \frac{\sin ax}{1 \pm \sin ax} dx &= \pm x + \frac{1}{a} \tan \left(\frac{\pi}{4} \mp \frac{ax}{2} \right). \\
334. \int \frac{dx}{(\sin ax)(1 \pm \sin ax)} &= \frac{1}{a} \tan \left(\frac{\pi}{4} \mp \frac{ax}{2} \right) + \frac{1}{a} \log \tan \frac{ax}{2}. \\
335. \int \frac{dx}{(1 + \sin ax)^2} &= -\frac{1}{2a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) - \frac{1}{6a} \tan^3 \left(\frac{\pi}{4} - \frac{ax}{2} \right).
\end{aligned}$$

$$\begin{aligned}
336. \int \frac{dx}{(1 - \sin ax)^2} &= \frac{1}{2a} \cot \left(\frac{\pi}{4} - \frac{ax}{2} \right) + \frac{1}{6a} \cot^3 \left(\frac{\pi}{4} - \frac{ax}{2} \right). \\
337. \int \frac{\sin ax}{(1 + \sin ax)^2} dx &= -\frac{1}{2a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) + \frac{1}{6a} \tan^3 \left(\frac{\pi}{4} - \frac{ax}{2} \right). \\
338. \int \frac{\sin ax}{(1 - \sin ax)^2} dx &= -\frac{1}{2a} \cot \left(\frac{\pi}{4} - \frac{ax}{2} \right) + \frac{1}{6a} \cot^3 \left(\frac{\pi}{4} - \frac{ax}{2} \right). \\
339. \int \frac{\sin x}{a + b \sin x} dx &= \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a + b \sin x}. \\
340. \int \frac{dx}{(\sin x)(a + b \sin x)} &= \frac{1}{a} \log \tan \frac{x}{2} - \frac{b}{a} \int \frac{dx}{a + b \sin x}. \\
341. \int \frac{dx}{(a + b \sin x)^2} &= \begin{cases} \frac{b \cos x}{(a^2 - b^2)(a + b \sin x)} + \frac{a}{a^2 - b^2} \int \frac{dx}{a + b \sin x}, \\ \text{or} \\ \frac{a \cos x}{(b^2 - a^2)(a + b \sin x)} + \frac{b}{b^2 - a^2} \int \frac{dx}{a + b \sin x}. \end{cases} \\
342. \int \frac{dx}{a^2 + b^2 \sin^2 cx} &= \frac{1}{ac\sqrt{a^2 + b^2}} \tan^{-1} \left(\frac{\sqrt{a^2 + b^2} \tan cx}{a} \right). \\
343. \int \frac{dx}{a^2 - b^2 \sin^2 cx} &= \begin{cases} \frac{1}{ac\sqrt{a^2 - b^2}} \tan^{-1} \left(\frac{\sqrt{a^2 - b^2} \tan cx}{a} \right), & a^2 > b^2, \\ \text{or} \\ \frac{1}{2ac\sqrt{b^2 - a^2}} \log \left(\frac{\sqrt{b^2 - a^2} \tan cx + a}{\sqrt{b^2 - a^2} \tan cx - a} \right), & a^2 < b^2. \end{cases} \\
344. \int \frac{\cos ax}{1 + \cos ax} dx &= x - \frac{1}{a} \tan \frac{ax}{2}. \\
345. \int \frac{\cos ax}{1 - \cos ax} dx &= -x - \frac{1}{a} \cot \frac{ax}{2}. \\
346. \int \frac{dx}{(\cos ax)(1 + \cos ax)} &= \frac{1}{a} \log \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right) - \frac{1}{a} \tan \frac{ax}{2}. \\
347. \int \frac{dx}{(\cos ax)(1 - \cos ax)} &= \frac{1}{a} \log \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right) - \frac{1}{a} \cot \frac{ax}{2}. \\
348. \int \frac{dx}{(1 + \cos ax)^2} &= \frac{1}{2a} \tan \frac{ax}{2} + \frac{1}{6a} \tan^3 \frac{ax}{2}. \\
349. \int \frac{dx}{(1 - \cos ax)^2} &= -\frac{1}{2a} \cot \frac{ax}{2} - \frac{1}{6a} \cot^3 \frac{ax}{2}. \\
350. \int \frac{\cos ax}{(1 + \cos ax)^2} dx &= \frac{1}{2a} \tan \frac{ax}{2} - \frac{1}{6a} \tan^3 \frac{ax}{2}. \\
351. \int \frac{\cos ax}{(1 - \cos ax)^2} dx &= \frac{1}{2a} \cot \frac{ax}{2} - \frac{1}{6a} \cot^3 \frac{ax}{2}. \\
352. \int \frac{\cos x}{a + b \cos x} dx &= \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a + b \cos x}. \\
353. \int \frac{dx}{(\cos x)(a + b \cos x)} &= \frac{1}{a} \log \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) - \frac{b}{a} \int \frac{dx}{a + b \cos x}. \\
354. \int \frac{dx}{(a + b \cos x)^2} &= \frac{b \sin x}{(b^2 - a^2)(a + b \cos x)} - \frac{a}{b^2 - a^2} \int \frac{dx}{a + b \cos x}. \\
355. \int \frac{\cos x}{(a + b \cos x)^2} dx &= \frac{a \sin x}{(a^2 - b^2)(a + b \cos x)} - \frac{b}{a^2 - b^2} \int \frac{dx}{a + b \cos x}.
\end{aligned}$$

$$\begin{aligned}
356. \int \frac{dx}{a^2 + b^2 - 2ab \cos cx} &= \frac{2}{c(a^2 - b^2)} \tan^{-1} \left(\frac{a+b}{a-b} \tan \frac{cx}{2} \right). \\
357. \int \frac{dx}{a^2 + b^2 \cos^2 cx} &= \frac{1}{ac\sqrt{a^2 + b^2}} \tan^{-1} \frac{a \tan cx}{\sqrt{a^2 + b^2}}. \\
358. \int \frac{dx}{a^2 - b^2 \cos^2 cx} &= \begin{cases} \frac{1}{ac\sqrt{a^2 - b^2}} \tan^{-1} \left(\frac{a \tan cx}{\sqrt{a^2 - b^2}} \right), & a^2 > b^2, \\ \text{or} \\ \frac{1}{2ac\sqrt{b^2 - a^2}} \log \left(\frac{a \tan cx - \sqrt{b^2 - a^2}}{a \tan cx + \sqrt{b^2 - a^2}} \right), & b^2 > a^2. \end{cases} \\
359. \int \frac{\sin ax}{1 \pm \cos ax} dx &= \mp \frac{1}{a} \log (1 \pm \cos ax). \\
360. \int \frac{\cos ax}{1 \pm \sin ax} dx &= \pm \frac{1}{a} \log (1 \pm \sin ax). \\
361. \int \frac{dx}{(\sin ax)(1 \pm \cos ax)} &= \pm \frac{1}{2a(1 \pm \cos ax)} + \frac{1}{2a} \log \tan \frac{ax}{2}. \\
362. \int \frac{dx}{(\cos ax)(1 \pm \sin ax)} &= \mp \frac{1}{2a(1 \pm \sin ax)} + \frac{1}{2a} \log \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right). \\
363. \int \frac{\sin ax}{(\cos ax)(1 \pm \cos ax)} dx &= \frac{1}{a} \log (\sec ax \pm 1). \\
364. \int \frac{\cos ax}{(\sin ax)(1 \pm \sin ax)} dx &= -\frac{1}{a} \log (\csc ax \pm 1). \\
365. \int \frac{\sin ax}{(\cos ax)(1 \pm \sin ax)} dx &= \frac{1}{2a(1 \pm \sin ax)} \pm \frac{1}{2a} \log \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right). \\
366. \int \frac{\cos ax}{(\sin ax)(1 \pm \cos ax)} dx &= -\frac{1}{2a(1 \pm \cos ax)} \pm \frac{1}{2a} \log \tan \frac{ax}{2}. \\
367. \int \frac{dx}{\sin ax \pm \cos ax} &= \frac{1}{a\sqrt{2}} \log \tan \left(\frac{ax}{2} \pm \frac{\pi}{8} \right). \\
368. \int \frac{dx}{(\sin ax \pm \cos ax)^2} &= \frac{1}{2a} \tan \left(ax \mp \frac{\pi}{4} \right). \\
369. \int \frac{dx}{1 + \cos ax \pm \sin ax} &= \pm \frac{1}{a} \log \left(1 \pm \tan \frac{ax}{2} \right). \\
370. \int \frac{dx}{a^2 \cos^2 cx - b^2 \sin^2 cx} &= \frac{1}{2abc} \log \left(\frac{b \tan cx + a}{b \tan cx - a} \right). \\
371. \int x \sin ax dx &= \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax. \\
372. \int x^2 \sin ax dx &= \frac{2x}{a^2} \sin ax + \frac{2 - a^2 x^2}{a^3} \cos ax. \\
373. \int x^3 \sin ax dx &= \frac{3a^2 x^2 - 6}{a^4} \sin ax + \frac{6x - a^2 x^3}{a^3} \cos ax. \\
374. \int x^m \sin ax dx &= \begin{cases} -\frac{1}{a} x^m \cos ax + \frac{m}{a} \int x^{m-1} \cos ax dx, \\ \text{or} \\ \cos ax \sum_{r=0}^{\lfloor \frac{m}{2} \rfloor} \frac{(-1)^{r+1} m!}{(m-2r)!} \frac{x^{m-2r}}{a^{2r+1}} + \sin ax \sum_{r=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{(-1)^r m!}{(m-2r-1)!} \frac{x^{m-2r-1}}{a^{2r+2}}. \end{cases}
\end{aligned}$$

$$\begin{aligned}
375. \int x \cos ax \, dx &= \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax. \\
376. \int x^2 \cos ax \, dx &= \frac{2x}{a^2} \cos ax + \frac{a^2 x^2 - 2}{a^3} \sin ax. \\
377. \int x^3 \cos ax \, dx &= \frac{3a^2 x^2 - 6}{a^4} \cos ax + \frac{a^2 x^3 - 6x}{a^3} \sin ax. \\
378. \int x^m \cos ax \, dx &= \begin{cases} \frac{x^m}{a} \sin ax - \frac{m}{a} \int x^{m-1} \sin ax \, dx, \\ \text{or} \\ \sin ax \sum_{r=0}^{\lfloor \frac{m}{2} \rfloor} \frac{(-1)^r m!}{(m-2r)!} \frac{x^{m-2r}}{a^{2r+1}} + \cos ax \sum_{r=0}^{\lfloor \frac{m-1}{2} \rfloor} \frac{(-1)^r m!}{(m-2r-1)!} \frac{x^{m-2r-1}}{a^{2r+2}}. \end{cases} \\
379. \int \frac{\sin ax}{x} \, dx &= \sum_{n=0}^{\infty} (-1)^n \frac{(ax)^{2n+1}}{(2n+1)(2n+1)!}. \\
380. \int \frac{\cos ax}{x} \, dx &= \sum_{n=0}^{\infty} (-1)^n \frac{(ax)^{2n}}{(2n)(2n)!}. \\
381. \int x \sin^2 ax \, dx &= \frac{x^2}{4} - \frac{x}{4a} \sin 2ax - \frac{1}{8a^2} \cos 2ax. \\
382. \int x^2 \sin^2 ax \, dx &= \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin 2ax - \frac{x}{4a^2} \cos 2ax. \\
383. \int x \sin^3 ax \, dx &= \frac{x}{12a} \cos 3ax - \frac{1}{36a^2} \sin 3ax - \frac{3x}{4a} \cos ax + \frac{3}{4a^2} \sin ax. \\
384. \int x \cos^2 ax \, dx &= \frac{x^2}{4} + \frac{x}{4a} \sin 2ax + \frac{1}{8a^2} \cos 2ax. \\
385. \int x^2 \cos^2 ax \, dx &= \frac{x^3}{6} + \left(\frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin 2ax + \frac{x}{4a^2} \cos 2ax. \\
386. \int x \cos^3 ax \, dx &= \frac{x}{12a} \sin 3ax + \frac{1}{36a^2} \cos 3ax + \frac{3x}{4a} \sin ax + \frac{3}{4a^2} \cos ax. \\
387. \int \frac{\sin ax}{x^m} \, dx &= \frac{\sin ax}{(1-m)x^{m-1}} + \frac{a}{m-1} \int \frac{\cos ax}{x^{m-1}} \, dx. \\
388. \int \frac{\cos ax}{x^m} \, dx &= \frac{\cos ax}{(1-m)x^{m-1}} + \frac{a}{1-m} \int \frac{\sin ax}{x^{m-1}} \, dx. \\
389. \int \frac{x}{1 \pm \sin ax} \, dx &= \mp \frac{x \cos ax}{a(1 \pm \sin ax)} + \frac{1}{a^2} \log(1 \pm \sin ax). \\
390. \int \frac{x}{1 + \cos ax} \, dx &= \frac{x}{a} \tan \frac{ax}{2} + \frac{2}{a^2} \log \cos \frac{ax}{2}. \\
391. \int \frac{x}{1 - \cos ax} \, dx &= -\frac{x}{a} \cot \frac{ax}{2} + \frac{2}{a^2} \log \sin \frac{ax}{2}. \\
392. \int \frac{x + \sin x}{1 + \cos x} \, dx &= x \tan \frac{x}{2}. \\
393. \int \frac{x - \sin x}{1 - \cos x} \, dx &= -x \cot \frac{x}{2}. \\
394. \int \sqrt{1 - \cos ax} \, dx &= -\frac{2 \sin ax}{a \sqrt{1 - \cos ax}} = -\frac{2\sqrt{2}}{a} \cos \frac{ax}{2}.
\end{aligned}$$

$$395. \int \sqrt{1 + \cos ax} dx = \frac{2 \sin ax}{a\sqrt{1 + \cos ax}} = \frac{2\sqrt{2}}{a} \sin \frac{ax}{2}.$$

For the following six integrals, each k represents an integer.

$$396. \int \sqrt{1 + \sin x} dx = \begin{cases} 2 \left(\sin \frac{x}{2} - \cos \frac{x}{2} \right), & (8k - 1)\frac{\pi}{2} < x \leq (8k + 3)\frac{\pi}{2}, \\ \text{or} \\ -2 \left(\sin \frac{x}{2} - \cos \frac{x}{2} \right), & (8k + 3)\frac{\pi}{2} < x \leq (8k + 7)\frac{\pi}{2}. \end{cases}$$

$$397. \int \sqrt{1 - \sin x} dx = \begin{cases} 2 \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right), & (8k - 3)\frac{\pi}{2} < x \leq (8k + 1)\frac{\pi}{2}, \\ \text{or} \\ -2 \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right), & (8k + 1)\frac{\pi}{2} < x \leq (8k + 5)\frac{\pi}{2}. \end{cases}$$

$$398. \int \frac{dx}{\sqrt{1 - \cos x}} = \begin{cases} \sqrt{2} \log \tan \frac{x}{4}, & 4k\pi < x \leq (4k + 2)\pi, \\ \text{or} \\ -\sqrt{2} \log \tan \frac{x}{4}, & (4k + 2)\pi < x \leq (4k + 4)\pi. \end{cases}$$

$$399. \int \frac{dx}{\sqrt{1 + \cos x}} = \begin{cases} \sqrt{2} \log \tan \left(\frac{x + \pi}{4} \right), & (4k - 1)\pi < x \leq (4k + 1)\pi, \\ \text{or} \\ -\sqrt{2} \log \tan \left(\frac{x + \pi}{4} \right), & (4k + 1)\pi < x \leq (4k + 3)\pi. \end{cases}$$

$$400. \int \frac{dx}{\sqrt{1 - \sin x}} = \begin{cases} \sqrt{2} \log \tan \left(\frac{x}{4} - \frac{\pi}{8} \right), & (8k + 1)\frac{\pi}{2} < x \leq (8k + 5)\frac{\pi}{2}, \\ \text{or} \\ -\sqrt{2} \log \tan \left(\frac{x}{4} - \frac{\pi}{8} \right), & (8k + 5)\frac{\pi}{2} < x \leq (8k + 9)\frac{\pi}{2}. \end{cases}$$

$$401. \int \frac{dx}{\sqrt{1 + \sin x}} = \begin{cases} \sqrt{2} \log \tan \left(\frac{x}{4} + \frac{\pi}{8} \right), & (8k - 1)\frac{\pi}{2} < x \leq (8k + 3)\frac{\pi}{2}, \\ \text{or} \\ -\sqrt{2} \log \tan \left(\frac{x}{4} + \frac{\pi}{8} \right), & (8k + 3)\frac{\pi}{2} < x \leq (8k + 7)\frac{\pi}{2}. \end{cases}$$

$$402. \int \tan^2 ax dx = \frac{1}{a} \tan ax - x.$$

$$403. \int \tan^3 ax dx = \frac{1}{2a} \tan^2 ax + \frac{1}{a} \log \cos ax.$$

$$404. \int \tan^4 ax dx = \frac{1}{3a} \tan^3 ax - \frac{1}{a} \tan ax + x.$$

$$405. \int \tan^n ax dx = \frac{1}{a(n-1)} \tan^{n-1} ax - \int \tan^{n-2} ax dx.$$

$$406. \int \cot^2 ax dx = -\frac{1}{a} \cot ax - x.$$

$$407. \int \cot^3 ax dx = -\frac{1}{2a} \cot^2 ax - \frac{1}{a} \log \sin ax.$$

$$408. \int \cot^4 ax dx = -\frac{1}{3a} \cot^3 ax + \frac{1}{a} \cot ax + x.$$

$$409. \int \cot^n ax dx = -\frac{1}{a(n-1)} \cot^{n-1} ax - \int \cot^{n-2} ax dx.$$

$$410. \int \frac{x}{\sin^2 ax} dx = \int x \csc^2 ax dx = -\frac{x \cot ax}{a} + \frac{1}{a^2} \log \sin ax.$$

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411. $\int \frac{x}{\sin^n ax} dx = \int x \csc^n ax dx = -\frac{x \cos ax}{a(n-1) \sin^{n-1} ax} - \frac{1}{a^2(n-1)(n-2) \sin^{n-2} ax} + \frac{n-2}{n-1} \int \frac{x}{\sin^{n-2} ax} dx.$
412. $\int \frac{x}{\cos^2 ax} dx = \int x \sec^2 ax dx = \frac{x}{a} \tan ax + \frac{1}{a^2} \log \cos ax.$
413. $\int \frac{x}{\cos^n ax} dx = \int x \sec^n ax dx = \frac{x \sin ax}{a(n-1) \cos^{n-1} ax} - \frac{1}{a^2(n-1)(n-2) \cos^{n-2} ax} + \frac{n-2}{n-1} \int \frac{x}{\cos^{n-2} ax} dx.$
414. $\int \frac{\sin ax}{\sqrt{1+b^2 \sin^2 ax}} dx = -\frac{1}{ab} \sin^{-1} \frac{b \cos ax}{\sqrt{1+b^2}}.$
415. $\int \frac{\sin ax}{\sqrt{1-b^2 \sin^2 ax}} dx = -\frac{1}{ab} \log \left(b \cos ax + \sqrt{1-b^2 \sin^2 ax} \right).$
416. $\int (\sin ax) \sqrt{1+b^2 \sin^2 ax} dx = -\frac{\cos ax}{2a} \sqrt{1+b^2 \sin^2 ax} - \frac{1+b^2}{2ab} \sin^{-1} \frac{b \cos ax}{\sqrt{1+b^2}}.$
417. $\int (\sin ax) \sqrt{1-b^2 \sin^2 ax} dx = -\frac{\cos ax}{2a} \sqrt{1-b^2 \sin^2 ax} - \frac{1-b^2}{2ab} \log \left(b \cos ax + \sqrt{1-b^2 \sin^2 ax} \right).$
418. $\int \frac{\cos ax}{\sqrt{1+b^2 \sin^2 ax}} dx = \frac{1}{ab} \log \left(b \sin ax + \sqrt{1+b^2 \sin^2 ax} \right).$
419. $\int \frac{\cos ax}{\sqrt{1-b^2 \sin^2 ax}} dx = \frac{1}{ab} \sin^{-1} (b \sin ax).$
420. $\int (\cos ax) \sqrt{1+b^2 \sin^2 ax} dx = \frac{\sin ax}{2a} \sqrt{1+b^2 \sin^2 ax} + \frac{1}{2ab} \log \left(b \sin ax + \sqrt{1+b^2 \sin^2 ax} \right).$
421. $\int (\cos ax) \sqrt{1-b^2 \sin^2 ax} dx = \frac{\sin ax}{2a} \sqrt{1-b^2 \sin^2 ax} + \frac{1}{2ab} \sin^{-1} (b \sin ax).$
- For the following integral, k represents an integer and $a > |b|$
422. $\int \frac{dx}{\sqrt{a+b \tan^2 cx}} = \begin{cases} \frac{1}{c\sqrt{a-b}} \sin^{-1} \left(\sqrt{\frac{a-b}{a}} \sin cx \right), & (4k-1)\frac{\pi}{2} < x \leq (4k+1)\frac{\pi}{2}, \\ \text{or} \\ \frac{-1}{c\sqrt{a-b}} \sin^{-1} \left(\sqrt{\frac{a-b}{a}} \sin cx \right), & (4k+1)\frac{\pi}{2} < x \leq (4k+3)\frac{\pi}{2}. \end{cases}$
423. $\int \cos^n x dx = \frac{1}{2^{n-1}} \sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{\sin[(n-2k)x]}{(n-2k)} + \frac{1}{2^n} \binom{n}{\frac{n}{2}} x, \quad n \text{ is an even integer.}$
424. $\int \cos^n x dx = \frac{1}{2^{n-1}} \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{\sin[(n-2k)x]}{(n-2k)}, \quad n \text{ is an odd integer.}$
425. $\int \sin^n x dx = \frac{1}{2^{n-1}} \sum_{k=0}^{\frac{n}{2}-1} \binom{n}{k} \frac{\sin \left([(n-2k)(\frac{\pi}{2} - x)] \right)}{(2k-n)} + \frac{1}{2^n} \binom{n}{\frac{n}{2}} x,$
 $n \text{ is an even integer.}$

$$426. \int \sin^n x \, dx = \frac{1}{2^{n-1}} \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \frac{\sin \left([(n-2k)(\frac{\pi}{2} - x)] \right)}{(2k-n)}, \quad n \text{ is an odd integer.}$$

5.4.17 FORMS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

$$427. \int \sin^{-1} ax \, dx = x \sin^{-1} ax + \frac{\sqrt{1-a^2x^2}}{a}.$$

$$428. \int \cos^{-1} ax \, dx = x \cos^{-1} ax - \frac{\sqrt{1-a^2x^2}}{a}.$$

$$429. \int \tan^{-1} ax \, dx = x \tan^{-1} ax - \frac{1}{2a} \log(1+a^2x^2).$$

$$430. \int \cot^{-1} ax \, dx = x \cot^{-1} ax + \frac{1}{2a} \log(1+a^2x^2).$$

$$431. \int \sec^{-1} ax \, dx = x \sec^{-1} ax - \frac{1}{a} \log(ax + \sqrt{a^2x^2-1}).$$

$$432. \int \csc^{-1} ax \, dx = x \csc^{-1} ax + \frac{1}{a} \log(ax + \sqrt{a^2x^2-1}).$$

$$433. \int \left(\sin^{-1} \frac{x}{a} \right) dx = x \sin^{-1} \frac{x}{a} + \sqrt{a^2-x^2}, \quad a > 0.$$

$$434. \int \left(\cos^{-1} \frac{x}{a} \right) dx = x \cos^{-1} \frac{x}{a} - \sqrt{a^2-x^2}, \quad a > 0.$$

$$435. \int \left(\tan^{-1} \frac{x}{a} \right) dx = x \tan^{-1} \frac{x}{a} - \frac{a}{2} \log(a^2+x^2).$$

$$436. \int \left(\cot^{-1} \frac{x}{a} \right) dx = x \cot^{-1} \frac{x}{a} + \frac{a}{2} \log(a^2+x^2).$$

$$437. \int x \sin^{-1}(ax) \, dx = \frac{1}{4a^2} \left((2a^2x^2-1) \sin^{-1}(ax) + ax \sqrt{1-a^2x^2} \right).$$

$$438. \int x \cos^{-1}(ax) \, dx = \frac{1}{4a^2} \left((2a^2x^2-1) \cos^{-1}(ax) - ax \sqrt{1-a^2x^2} \right).$$

$$439. \int x^n \sin^{-1}(ax) \, dx = \frac{x^{n+1}}{n+1} \sin^{-1}(ax) - \frac{a}{n+1} \int \frac{x^{n+1}}{\sqrt{1-a^2x^2}} dx, \quad n \neq -1.$$

$$440. \int x^n \cos^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cos^{-1}(ax) + \frac{a}{n+1} \int \frac{x^{n+1}}{\sqrt{1-a^2x^2}} dx, \quad n \neq -1.$$

$$441. \int x \tan^{-1}(ax) \, dx = \frac{1+a^2x^2}{2a^2} \tan^{-1}(ax) - \frac{x}{2a}.$$

$$442. \int x^n \tan^{-1}(ax) \, dx = \frac{x^{n+1}}{n+1} \tan^{-1}(ax) - \frac{a}{n+1} \int \frac{x^{n+1}}{1+a^2x^2} dx.$$

$$443. \int x \cot^{-1}(ax) \, dx = \frac{1+a^2x^2}{2a^2} \cot^{-1}(ax) + \frac{x}{2a}.$$

$$444. \int x^n \cot^{-1}(ax) \, dx = \frac{x^{n+1}}{n+1} \cot^{-1}(ax) + \frac{a}{n+1} \int \frac{x^{n+1}}{1+a^2x^2} dx.$$

$$445. \int \frac{\sin^{-1}(ax)}{x^2} dx = a \log \left(\frac{1-\sqrt{1-a^2x^2}}{x} \right) - \frac{\sin^{-1}(ax)}{x}.$$

$$446. \int \frac{\cos^{-1}(ax)}{x^2} dx = -\frac{1}{x} \cos^{-1}(ax) + a \log \left(\frac{1+\sqrt{1-a^2x^2}}{x} \right).$$

447. $\int \frac{\tan^{-1}(ax)}{x^2} dx = -\frac{1}{x} \tan^{-1}(ax) - \frac{a}{2} \log \left(\frac{1+a^2x^2}{x^2} \right).$

448. $\int \frac{\cot^{-1}(ax)}{x^2} dx = -\frac{1}{x} \cot^{-1}(ax) - \frac{a}{2} \log \left(\frac{x^2}{1+a^2x^2} \right).$

449. $\int (\sin^{-1}(ax))^2 dx = x(\sin^{-1}(ax))^2 - 2x + \frac{2\sqrt{1-a^2x^2}}{a} \sin^{-1}(ax).$

450. $\int (\cos^{-1}(ax))^2 dx = x(\cos^{-1}(ax))^2 - 2x - \frac{2\sqrt{1-a^2x^2}}{a} \cos^{-1}(ax).$

451. $\int (\sin^{-1}(ax))^n dx =$

$$\begin{cases} x(\sin^{-1}(ax))^n + \frac{n\sqrt{1-a^2x^2}}{a}(\sin^{-1}(ax))^{n-1} - n(n-1) \int (\sin^{-1}(ax))^{n-2} dx, \\ \text{or} \\ \sum_{r=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^r n!}{(n-2r)!} x(\sin^{-1} ax)^{n-2r} + \sum_{r=0}^{\lfloor \frac{n-1}{2} \rfloor} (-1)^r \frac{n! \sqrt{1-a^2x^2}}{(n-2r-1)!a} (\sin^{-1} ax)^{n-2r-1}. \end{cases}$$

452. $\int (\cos^{-1}(ax))^n dx =$

$$\begin{cases} x(\cos^{-1}(ax))^n - \frac{n\sqrt{1-a^2x^2}}{a}(\cos^{-1}(ax))^{n-1} - n(n-1) \int (\cos^{-1}(ax))^{n-2} dx, \\ \text{or} \\ \sum_{r=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^r n!}{(n-2r)!} x(\cos^{-1} ax)^{n-2r} - \sum_{r=0}^{\lfloor \frac{n-1}{2} \rfloor} (-1)^r \frac{n! \sqrt{1-a^2x^2}}{(n-2r-1)!a} (\cos^{-1} ax)^{n-2r-1}. \end{cases}$$

453. $\int \frac{\sin^{-1} ax}{\sqrt{1-a^2x^2}} dx = \frac{1}{2a} (\sin^{-1} ax)^2.$

454. $\int \frac{x^n \sin^{-1} ax}{\sqrt{1-a^2x^2}} dx = -\frac{x^{n-1}}{na^2} \sqrt{1-a^2x^2} \sin^{-1} ax + \frac{x^n}{n^2a}$

$$+ \frac{n-1}{na^2} \int \frac{x^{n-2} \sin^{-1} ax}{\sqrt{1-a^2x^2}} dx.$$

455. $\int \frac{\cos^{-1} ax}{\sqrt{1-a^2x^2}} dx = -\frac{1}{2a} (\cos^{-1} ax)^2.$

456. $\int \frac{x^n \cos^{-1} ax}{\sqrt{1-a^2x^2}} dx = -\frac{x^{n-1}}{na^2} \sqrt{1-a^2x^2} \cos^{-1} ax - \frac{x^n}{n^2a}$

$$+ \frac{n-1}{na^2} \int \frac{x^{n-2} \cos^{-1} ax}{\sqrt{1-a^2x^2}} dx.$$

457. $\int \frac{\tan^{-1} ax}{1+a^2x^2} dx = \frac{1}{2a} (\tan^{-1} ax)^2.$

458. $\int \frac{\cot^{-1} ax}{1+a^2x^2} dx = -\frac{1}{2a} (\cot^{-1} ax)^2.$

459. $\int x \sec^{-1} ax dx = \frac{x^2}{2} \sec^{-1} ax - \frac{1}{2a^2} \sqrt{a^2x^2-1}.$

460. $\int x^n \sec^{-1} ax dx = \frac{x^{n+1}}{n+1} \sec^{-1} ax - \frac{1}{n+1} \int \frac{x^n}{\sqrt{a^2x^2-1}} dx.$

461. $\int \frac{\sec^{-1} ax}{x^2} dx = -\frac{\sec^{-1} ax}{x} + \frac{\sqrt{a^2x^2-1}}{x}.$

$$\begin{aligned}
462. \int x \csc^{-1} ax \, dx &= \frac{x^2}{2} \csc^{-1} ax + \frac{1}{2a^2} \sqrt{a^2 x^2 - 1}. \\
463. \int x^n \csc^{-1} ax \, dx &= \frac{x^{n+1}}{n+1} \csc^{-1} ax + \frac{1}{n+1} \int \frac{x^n}{\sqrt{a^2 x^2 - 1}} \, dx. \\
464. \int \frac{\csc^{-1} ax}{x^2} \, dx &= -\frac{\csc^{-1} ax}{x} - \frac{\sqrt{a^2 x^2 - 1}}{x}.
\end{aligned}$$

5.4.18 LOGARITHMIC FORMS

$$\begin{aligned}
465. \int \log x \, dx &= x \log x - x. \\
466. \int x \log x \, dx &= \frac{x^2}{2} \log x - \frac{x^2}{4}. \\
467. \int x^2 \log x \, dx &= \frac{x^3}{3} \log x - \frac{x^3}{9}. \\
468. \int x^n \log x \, dx &= \frac{x^{n+1}}{n+1} \log x - \frac{x^{n+1}}{(n+1)^2}. \\
469. \int (\log x)^2 \, dx &= x(\log x)^2 - 2x \log x + 2x. \\
470. \int (\log x)^n \, dx &= \begin{cases} x(\log x)^n - n \int (\log x)^{n-1} \, dx, & n \neq -1, \\ \text{or} \\ (-1)^n n! x \sum_{r=0}^n \frac{(-\log x)^r}{r!}, & n \neq -1. \end{cases} \\
471. \int \frac{(\log x)^n}{x} \, dx &= \frac{1}{n+1} (\log x)^{n+1}, \quad n \neq -1. \\
472. \int \frac{dx}{\log x} &= \log(\log x) + \log x + \frac{(\log x)^2}{2 \cdot 2!} + \frac{(\log x)^3}{3 \cdot 3!} + \dots \\
473. \int \frac{dx}{x \log x} &= \log(\log x). \\
474. \int \frac{dx}{x(\log x)^n} &= \frac{1}{(1-n)(\log x)^{n-1}}, \quad n \neq 1. \\
475. \int \frac{x^m dx}{(\log x)^n} &= \frac{x^{m+1}}{(1-n)(\log x)^{n-1}} + \frac{m+1}{n-1} \int \frac{x^m dx}{(\log x)^{n-1}}, \quad n \neq 1. \\
476. \int x^m (\log x)^n \, dx &= \begin{cases} \frac{x^{m+1}(\log x)^n}{m+1} - \frac{n}{m+1} \int x^m (\log x)^{n-1} \, dx, \\ \text{or} \\ (-1)^n \frac{n!}{m+1} x^{m+1} \sum_{r=0}^n \frac{(-\log x)^r}{r!(m+1)^{n-r}}. \end{cases} \\
477. \int x^p \cos(b \log x) \, dx &= \frac{x^{p+1}}{(p+1)^2 + b^2} [b \sin(b \log x) + (p+1) \cos(b \log x)]. \\
478. \int x^p \sin(b \log x) \, dx &= \frac{x^{p+1}}{(p+1)^2 + b^2} [(p+1) \sin(b \log x) - b \cos(b \log x)]. \\
479. \int \log(ax+b) \, dx &= \frac{ax+b}{a} \log(ax+b) - x.
\end{aligned}$$

480. $\int \frac{\log(ax+b)}{x^2} dx = \frac{a}{b} \log x - \frac{ax+b}{bx} \log(ax+b).$

481. $\int x^m \log(ax+b) dx = \frac{1}{m+1} \left[x^{m+1} - \left(-\frac{b}{a}\right)^{m+1} \right] \log(ax+b) - \frac{1}{m+1} \left(-\frac{b}{a}\right)^{m+1} \sum_{r=1}^{m+1} \frac{1}{r} \left(-\frac{ax}{b}\right)^r.$

482. $\int \frac{\log(ax+b)}{x^m} dx = -\frac{1}{m-1} \frac{\log(ax+b)}{x^{m-1}} + \frac{1}{m-1} \left(-\frac{a}{b}\right)^{m-1} \log \frac{ax+b}{x} + \frac{1}{m-1} \left(-\frac{a}{b}\right)^{m-1} \sum_{r=1}^{m-2} \frac{1}{r} \left(-\frac{b}{ax}\right)^r, \quad m > 2.$

483. $\int \log \frac{x+a}{x-a} dx = (x+a) \log(x+a) - (x-a) \log(x-a).$

484. $\int x^m \log \frac{x+a}{x-a} dx = \frac{x^{m+1} - (-a)^{m+1}}{m+1} \log(x+a) - \frac{x^{m+1} - a^{m+1}}{m+1} \log(x-a) + \frac{2a^{m+1}}{m+1} \sum_{r=1}^{\lfloor \frac{m+1}{2} \rfloor} \frac{1}{m-2r+2} \left(\frac{x}{a}\right)^{m-2r+2}.$

485. $\int \frac{1}{x^2} \log \frac{x+a}{x-a} dx = \frac{1}{x} \log \frac{x-a}{x+a} - \frac{1}{a} \log \frac{x^2-a^2}{x^2}.$

For the following two integrals, $X = a + bx + cx^2$.

486. $\int \log X dx = \begin{cases} \left(x + \frac{b}{2c}\right) \log X - 2x + \frac{\sqrt{4ac-b^2}}{c} \tan^{-1} \frac{2cx+b}{\sqrt{4ac-b^2}}, & b^2 - 4ac < 0, \\ \text{or} \\ \left(x + \frac{b}{2c}\right) \log X - 2x + \frac{\sqrt{b^2-4ac}}{c} \tanh^{-1} \frac{2cx+b}{\sqrt{b^2-4ac}}, & b^2 - 4ac > 0. \end{cases}$

487. $\int x^n \log X dx = \frac{x^{n+1}}{n+1} \log X - \frac{2c}{n+1} \int \frac{x^{n+2}}{X} dx - \frac{b}{n+1} \int \frac{x^{n+1}}{X} dx, \quad n \neq -1.$

488. $\int \log(x^2 + a^2) dx = x \log(x^2 + a^2) - 2x + 2a \tan^{-1} \frac{x}{a}.$

489. $\int \log(x^2 - a^2) dx = x \log(x^2 - a^2) - 2x + a \log \frac{x+a}{x-a}.$

490. $\int x \log(x^2 + a^2) dx = \frac{1}{2} (x^2 + a^2) \log(x^2 + a^2) - \frac{1}{2} x^2.$

491. $\int \log(x + \sqrt{x^2 \pm a^2}) dx = x \log(x + \sqrt{x^2 \pm a^2}) - \sqrt{x^2 \pm a^2}.$

492. $\int x \log(x + \sqrt{x^2 \pm a^2}) dx = \left(\frac{x^2}{2} \pm \frac{a^2}{4}\right) \log(x + \sqrt{x^2 \pm a^2}) - \frac{x\sqrt{x^2 \pm a^2}}{4}.$

493. $\int x^m \log(x + \sqrt{x^2 \pm a^2}) dx = \frac{x^{m+1}}{m+1} \log(x + \sqrt{x^2 \pm a^2}) - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2 \pm a^2}} dx.$

494. $\int \frac{\log(x + \sqrt{x^2 + a^2})}{x^2} dx = -\frac{\log(x + \sqrt{x^2 + a^2})}{x} - \frac{1}{a} \log \frac{a + \sqrt{x^2 + a^2}}{x}.$

$$495. \int \frac{\log(x + \sqrt{x^2 - a^2})}{x^2} dx = -\frac{\log(x + \sqrt{x^2 - a^2})}{x} + \frac{1}{|a|} \sec^{-1} \frac{x}{a}.$$

$$496. \int x^n \log(x^2 - a^2) dx = \frac{1}{n+1} \left[x^{n+1} \log(x^2 - a^2) - a^{n+1} \log(x - a) \right. \\ \left. - (-a)^{n+1} \log(x + a) - 2 \sum_{r=0}^{\lfloor \frac{n}{2} \rfloor} \frac{a^{2r} x^{n-2r+1}}{n-2r+1} \right].$$

5.4.19 EXPONENTIAL FORMS

$$497. \int e^x dx = e^x.$$

$$498. \int e^{-x} dx = -e^{-x}.$$

$$499. \int e^{ax} dx = \frac{e^{ax}}{a}.$$

$$500. \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1).$$

$$501. \int x^m e^{ax} dx = \begin{cases} \frac{x^m e^{ax}}{a} - \frac{m}{a} \int x^{m-1} e^{ax} dx, \\ \text{or} \\ e^{ax} \sum_{r=0}^m (-1)^r \frac{m! x^{m-r}}{(m-r)! a^{r+1}}. \end{cases}$$

$$502. \int \frac{e^{ax}}{x} dx = \log x + \frac{ax}{1!} + \frac{a^2 x^2}{2 \cdot 2!} + \frac{a^3 x^3}{3 \cdot 3!} + \dots$$

$$503. \int \frac{e^{ax}}{x^m} dx = \frac{1}{1-m} \frac{e^{ax}}{x^{m-1}} + \frac{a}{m-1} \int \frac{e^{ax}}{x^{m-1}} dx, \quad m \neq 1.$$

$$504. \int e^{ax} \log x dx = \frac{e^{ax} \log x}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx.$$

$$505. \int \frac{dx}{1+e^x} = x - \log(1+e^x) = \log \frac{e^x}{1+e^x}.$$

$$506. \int \frac{dx}{a+be^{px}} = \frac{x}{a} - \frac{1}{ap} \log(a+be^{px}).$$

$$507. \int \frac{dx}{ae^{mx} + be^{-mx}} = \frac{1}{m\sqrt{ab}} \tan^{-1} \left(e^{mx} \sqrt{\frac{a}{b}} \right), \quad a > 0, b > 0.$$

$$508. \int \frac{dx}{ae^{mx} - be^{-mx}} = \begin{cases} \frac{1}{2m\sqrt{ab}} \log \left(\frac{\sqrt{a}e^{mx} - \sqrt{b}}{\sqrt{a}e^{mx} + \sqrt{b}} \right), & a > 0, b > 0, \\ \text{or} \\ \frac{-1}{m\sqrt{ab}} \tanh^{-1} \left(\sqrt{\frac{a}{b}} e^{mx} \right), & a > 0, b > 0. \end{cases}$$

$$509. \int (a^x - a^{-x}) dx = \frac{a^x + a^{-x}}{\log a}.$$

$$510. \int \frac{e^{ax}}{b+ce^{ax}} dx = \frac{1}{ac} \log(b+ce^{ax}).$$

$$511. \int \frac{xe^{ax}}{(1+ax)^2} dx = \frac{e^{ax}}{a^2(1+ax)}.$$

$$\begin{aligned}
512. \int x e^{-x^2} dx &= -\frac{1}{2} e^{-x^2}. \\
513. \int e^{ax} \sin(bx) dx &= \frac{e^{ax} [a \sin(bx) - b \cos(bx)]}{a^2 + b^2}. \\
514. \int e^{ax} \sin(bx) \sin(cx) dx &= \frac{e^{ax} [(b-c) \sin(b-c)x + a \cos(b-c)x]}{2[a^2 + (b-c)^2]} \\
&\quad - \frac{e^{ax} [(b+c) \sin(b+c)x + a \cos(b+c)x]}{2[a^2 + (b+c)^2]}. \\
515. \int e^{ax} \sin(bx) \cos(cx) dx &= \frac{e^{ax} [a \sin(b-c)x - (b-c) \cos(b-c)x]}{2[a^2 + (b-c)^2]} \\
&\quad + \frac{e^{ax} [a \sin(b+c)x - (b+c) \cos(b+c)x]}{2[a^2 + (b+c)^2]}. \\
516. \int e^{ax} \sin(bx) \sin(bx+c) dx &= \frac{e^{ax} \cos c}{2a} - \frac{e^{ax} [a \cos 2bx + c + 2b \sin 2bx + c]}{2[a^2 + 4b^2]}. \\
517. \int e^{ax} \sin(bx) \cos(bx+c) dx &= -\frac{e^{ax} \sin c}{2a} + \frac{e^{ax} [a \sin 2bx + c - 2b \cos 2bx + c]}{2[a^2 + 4b^2]}. \\
518. \int e^{ax} \cos(bx) dx &= \frac{e^{ax}}{a^2 + b^2} [a \cos(bx) + b \sin(bx)]. \\
519. \int e^{ax} \cos(bx) \cos(cx) dx &= \frac{e^{ax} [(b-c) \sin(b-c)x + a \cos(b-c)x]}{2[a^2 + (b-c)^2]} \\
&\quad + \frac{e^{ax} [(b+c) \sin(b+c)x + a \cos(b+c)x]}{2[a^2 + (b+c)^2]}. \\
520. \int e^{ax} \cos(bx) \cos(bx+c) dx &= \frac{e^{ax} \cos c}{2a} + \frac{e^{ax} [a \cos 2bx + c + 2b \sin 2bx + c]}{2[a^2 + 4b^2]}. \\
521. \int e^{ax} \cos(bx) \sin(bx+c) dx &= \frac{e^{ax} \sin c}{2a} + \frac{e^{ax} [a \sin 2bx + c - 2b \cos 2bx + c]}{2[a^2 + 4b^2]}. \\
522. \int e^{ax} \sin^n(bx) dx &= \frac{1}{a^2 + n^2 b^2} [(a \sin(bx) - nb \cos(bx)) e^{ax} \sin^{n-1}(bx) \\
&\quad + n(n-1)b^2 \int e^{ax} \sin^{n-2}(bx) dx]. \\
523. \int e^{ax} \cos^n(bx) dx &= \frac{1}{a^2 + n^2 b^2} [(a \cos(bx) + nb \sin(bx)) e^{ax} \cos^{n-1}(bx) \\
&\quad + n(n-1)b^2 \int e^{ax} \cos^{n-2}(bx) dx]. \\
524. \int x^m e^x \sin x dx &= \frac{1}{2} x^m e^x (\sin x - \cos x) - \frac{m}{2} \int x^{m-1} e^x \sin x dx \\
&\quad + \frac{m}{2} \int x^{m-1} e^x \cos x dx. \\
525. \int x^m e^{ax} \sin bx dx &= x^m e^{ax} \frac{a \sin(bx) - b \cos(bx)}{a^2 + b^2} \\
&\quad - \frac{m}{a^2 + b^2} \int x^{m-1} e^{ax} (a \sin(bx) - b \cos(bx)) dx. \\
526. \int x^m e^x \cos x dx &= \frac{1}{2} x^m e^x (\sin x + \cos x) - \frac{m}{2} \int x^{m-1} e^x \sin x dx \\
&\quad - \frac{m}{2} \int x^{m-1} e^x \cos x dx. \\
527. \int x^m e^{ax} \cos bx dx &= x^m e^{ax} \frac{a \cos(bx) + b \sin(bx)}{a^2 + b^2} \\
&\quad - \frac{m}{a^2 + b^2} \int x^{m-1} e^{ax} (a \cos(bx) + b \sin(bx)) dx.
\end{aligned}$$

528. $\int e^{ax} \cos^m x \sin^n x dx =$

$$\left\{ \begin{array}{l} \frac{e^{ax} (\cos^{m-1} x) (\sin^n x) [a \cos x + (m+n) \sin x]}{(m+n)^2 + a^2} \\ \quad - \frac{na}{(m+n)^2 + a^2} \int e^{ax} (\cos^{m-1} x) (\sin^{n-1} x) dx \\ \quad + \frac{(m-1)(m+n)}{(m+n)^2 + a^2} \int e^{ax} (\cos^{m-2} x) (\sin^n x) dx, \\ \text{or} \\ \frac{e^{ax} (\cos^m x) (\sin^{n-1} x) [a \sin x - (m+n) \cos x]}{(m+n)^2 + a^2} \\ \quad + \frac{ma}{(m+n)^2 + a^2} \int e^{ax} (\cos^{m-1} x) (\sin^{n-1} x) dx \\ \quad + \frac{(n-1)(m+n)}{(m+n)^2 + a^2} \int e^{ax} (\cos^m x) (\sin^{n-2} x) dx, \\ \text{or} \\ \frac{e^{ax} (\cos^{m-1} x) (\sin^{n-1} x) [a \sin x \cos x + m \sin^2 x - n \cos^2 x]}{(m+n)^2 + a^2} \\ \quad + \frac{m(m-1)}{(m+n)^2 + a^2} \int e^{ax} (\cos^{m-2} x) (\sin^n x) dx \\ \quad + \frac{n(n-1)}{(m+n)^2 + a^2} \int e^{ax} (\cos^m x) (\sin^{n-2} x) dx, \\ \text{or} \\ \frac{e^{ax} (\cos^{m-1} x) (\sin^{n-1} x) [a \sin x \cos x + m \sin^2 x - n \cos^2 x]}{(m+n)^2 + a^2} \\ \quad + \frac{m(m-1)}{(m+n)^2 + a^2} \int e^{ax} (\cos^{m-2} x) (\sin^{n-2} x) dx \\ \quad + \frac{(n-m)(n+m-1)}{(m+n)^2 + a^2} \int e^{ax} (\cos^m x) (\sin^{n-2} x) dx. \end{array} \right.$$

529. $\int x e^{ax} \sin (bx) dx = \frac{x e^{ax}}{a^2 + b^2} [a \sin (bx) - b \cos (bx)]$

$$- \frac{e^{ax}}{(a^2 + b^2)^2} [(a^2 - b^2) \sin bx - 2ab \cos (bx)].$$

530. $\int x e^{ax} \cos (bx) dx = \frac{x e^{ax}}{a^2 + b^2} [a \cos (bx) + b \sin (bx)]$

$$- \frac{e^{ax}}{(a^2 + b^2)^2} [(a^2 - b^2) \cos bx + 2ab \sin (bx)].$$

531. $\int \frac{e^{ax}}{\sin^n x} dx = -\frac{e^{ax} [a \sin x + (n-2) \cos x]}{(n-1)(n-2) \sin^{n-1} x} + \frac{a^2 + (n-2)^2}{(n-1)(n-2)} \int \frac{e^{ax}}{\sin^{n-2} x} dx.$

532. $\int \frac{e^{ax}}{\cos^n x} dx = -\frac{e^{ax} [a \cos x - (n-2) \sin x]}{(n-1)(n-2) \cos^{n-1} x} + \frac{a^2 + (n-2)^2}{(n-1)(n-2)} \int \frac{e^{ax}}{\cos^{n-2} x} dx.$

533. $\int e^{ax} \tan^n x dx = e^{ax} \frac{\tan^{n-1} x}{n-1} - \frac{a}{n-1} \int e^{ax} \tan^{n-1} x dx - \int e^{ax} \tan^{n-2} x dx.$

5.4.20 HYPERBOLIC FORMS

534. $\int \sinh x dx = \cosh x.$

535. $\int \cosh x dx = \sinh x.$

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536. $\int \tanh x \, dx = \log \cosh x.$
537. $\int \coth x \, dx = \log \sinh x.$
538. $\int \operatorname{sech} x \, dx = \tan^{-1}(\sinh x).$
539. $\int \operatorname{csch} x \, dx = \log \tanh \left(\frac{x}{2} \right).$
540. $\int x \sinh x \, dx = x \cosh x - \sinh x.$
541. $\int x^n \sinh x \, dx = x^n \cosh x - n \int x^{n-1} (\cosh x) \, dx.$
542. $\int x \cosh x \, dx = x \sinh x - \cosh x.$
543. $\int x^n \cosh x \, dx = x^n \sinh x - n \int x^{n-1} (\sinh x) \, dx.$
544. $\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x.$
545. $\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x.$
546. $\int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2}.$
547. $\int \sinh^m x \cosh^n x \, dx =$

$$\begin{cases} \frac{1}{m+n} \sinh^{m+1} x \cosh^{n-1} x + \frac{n-1}{m+n} \int \sinh^m x \cosh^{n-2} x \, dx, & m+n \neq 0, \\ \text{or} \\ \frac{1}{m+n} \sinh^{m-1} x \cosh^{n+1} x - \frac{m-1}{m+n} \int \sinh^{m-2} x \cosh^n x \, dx, & m+n \neq 0. \end{cases}$$
548. $\int \frac{dx}{(\sinh^m x)(\cosh^n x)} =$

$$\begin{cases} -\frac{1}{(m-1)(\sinh^{m-1} x)(\cosh^{n-1} x)} - \frac{m+n-2}{m-1} \int \frac{dx}{(\sinh^{m-2} x)(\cosh^n x)} \, dx, & m \neq 1, \\ \text{or} \\ \frac{1}{(n-1)(\sinh^{m-1} x)(\cosh^{n-1} x)} + \frac{m+n-2}{n-1} \int \frac{dx}{(\sinh^m x)(\cosh^{n-2} x)} \, dx, & n \neq 1. \end{cases}$$
549. $\int \tanh^2 x \, dx = x - \tanh x.$
550. $\int \tanh^n x \, dx = -\frac{\tanh^{n-1} x}{n-1} + \int (\tanh^{n-2} x) \, dx, \quad n \neq 1.$
551. $\int \operatorname{sech}^2 x \, dx = \tanh x.$
552. $\int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2}.$
553. $\int \coth^2 x \, dx = x - \coth x.$
554. $\int \coth^n x \, dx = -\frac{\coth^{n-1} x}{n-1} + \int \coth^{n-2} x \, dx, \quad n \neq 1.$

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555. $\int \operatorname{csch}^2 x \, dx = -\coth x.$
556. $\int (\sinh mx)(\sinh nx) \, dx = \frac{\sinh(m+n)x}{2(m+n)} - \frac{\sinh(m-n)x}{2(m-n)}, \quad m^2 \neq n^2.$
557. $\int (\cosh mx)(\cosh nx) \, dx = \frac{\sinh(m+n)x}{2(m+n)} + \frac{\sinh(m-n)x}{2(m-n)}, \quad m^2 \neq n^2.$
558. $\int (\sinh mx)(\cosh nx) \, dx = \frac{\cosh(m+n)x}{2(m+n)} + \frac{\cosh(m-n)x}{2(m-n)}, \quad m^2 \neq n^2.$
559. $\int \left(\sinh^{-1} \frac{x}{a} \right) dx = x \sinh^{-1} \frac{x}{a} - \sqrt{x^2 + a^2}, \quad a > 0.$
560. $\int x \left(\sinh^{-1} \frac{x}{a} \right) dx = \left(\frac{x^2}{2} + \frac{a^2}{4} \right) \sinh^{-1} \frac{x}{a} - \frac{x}{4} \sqrt{x^2 + a^2}, \quad a > 0.$
561. $\int x^n \sinh^{-1} x \, dx = \frac{x^{n+1}}{n+1} \sinh^{-1} x - \frac{1}{n+1} \int \frac{x^{n+1}}{\sqrt{1+x^2}} dx, \quad n \neq -1.$
562. $\int^z \cosh^{-1} \frac{x}{a} dx = \begin{cases} z \cosh^{-1} \frac{z}{a} - \sqrt{z^2 - a^2}, & \cosh^{-1} \frac{z}{a} > 0, \\ \text{or} \\ z \cosh^{-1} \frac{z}{a} + \sqrt{z^2 - a^2}, & \cosh^{-1} \frac{z}{a} < 0, \quad a > 0. \end{cases}$
563. $\int x \left(\cosh^{-1} \frac{x}{a} \right) dx = \left(\frac{x^2}{2} - \frac{a^2}{4} \right) \cosh^{-1} \frac{x}{a} - \frac{x}{4} \sqrt{x^2 - a^2}.$
564. $\int x^n \cosh^{-1} x \, dx = \frac{x^{n+1}}{n+1} \cosh^{-1} x - \frac{1}{n+1} \int \frac{x^{n+1}}{\sqrt{x^2 - 1}} dx, \quad n \neq -1.$
565. $\int \left(\tanh^{-1} \frac{x}{a} \right) dx = x \tanh^{-1} \frac{x}{a} + \frac{a}{2} \log(a^2 - x^2), \quad \left| \frac{x}{a} \right| < 1.$
566. $\int \left(\coth^{-1} \frac{x}{a} \right) dx = x \coth^{-1} \frac{x}{a} + \frac{a}{2} \log(x^2 - a^2), \quad \left| \frac{x}{a} \right| > 1.$
567. $\int x \left(\tanh^{-1} \frac{x}{a} \right) dx = \frac{x^2 - a^2}{2} \tanh^{-1} \frac{x}{a} + \frac{ax}{2}, \quad \left| \frac{x}{a} \right| < 1.$
568. $\int x^n \tanh^{-1} x \, dx = \frac{x^{n+1}}{n+1} \tanh^{-1} x - \frac{1}{n+1} \int \frac{x^{n+1}}{1-x^2} dx, \quad n \neq -1.$
569. $\int x \left(\coth^{-1} \frac{x}{a} \right) dx = \frac{x^2 - a^2}{2} \coth^{-1} \frac{x}{a} + \frac{ax}{2}, \quad \left| \frac{x}{a} \right| > 1.$
570. $\int x^n \coth^{-1} x \, dx = \frac{x^{n+1}}{n+1} \coth^{-1} x + \frac{1}{n+1} \int \frac{x^{n+1}}{x^2 - 1} dx, \quad n \neq -1.$
571. $\int \operatorname{sech}^{-1} x \, dx = x \operatorname{sech}^{-1} x + \sin^{-1} x.$
572. $\int x \operatorname{sech}^{-1} x \, dx = \frac{x^2}{2} \operatorname{sech}^{-1} x - \frac{1}{2} \sqrt{1 - x^2}.$
573. $\int x^n \operatorname{sech}^{-1} x \, dx = \frac{x^{n+1}}{n+1} \operatorname{sech}^{-1} x + \frac{1}{n+1} \int \frac{x^n}{\sqrt{1-x^2}} dx, \quad n \neq -1.$
574. $\int \operatorname{csch}^{-1} x \, dx = x \operatorname{csch}^{-1} x + \frac{x}{|x|} \sinh^{-1} x.$
575. $\int x \operatorname{csch}^{-1} x \, dx = \frac{x^2}{2} \operatorname{csch}^{-1} x + \frac{1}{2} \frac{x}{|x|} \sqrt{1 + x^2}.$

$$576. \int x^n \operatorname{csch}^{-1} x \, dx = \frac{x^{n+1}}{n+1} \operatorname{csch}^{-1} x + \frac{1}{n+1} \frac{x}{|x|} \int \frac{x^n}{\sqrt{1+x^2}} \, dx, \quad n \neq -1.$$

5.4.21 BESSEL FUNCTIONS

$Z_p(x)$ represents any of the Bessel functions $\{J_p(x), Y_p(x), K_p(x), I_p(x)\}$.

$$577. \int x^{p+1} Z_p(x) \, dx = x^{p+1} Z_{p+1}(x).$$

$$578. \int x^{-p+1} Z_p(x) \, dx = -x^{-p+1} Z_{p-1}(x).$$

$$579. \int x [Z_p(ax)]^2 \, dx = \frac{x^2}{2} [[Z_p(ax)]^2 - Z_{p-1}(ax) Z_{p+1}(ax)].$$

$$580. \int Z_1(x) \, dx = -Z_0(x).$$

$$581. \int x Z_0(x) \, dx = x Z_1(x).$$

5.5 TABLE OF DEFINITE INTEGRALS

$$582. \int_0^\infty x^{n-1} e^{-x} \, dx = \Gamma(n), \quad \operatorname{Re} n > 0.$$

$$583. \int_0^\infty x^n p^{-x} \, dx = \frac{n!}{(\log p)^{n+1}}, \quad p > 0, \quad n \text{ is a non-negative integer.}$$

$$584. \int_0^\infty x^{n-1} e^{-(a+1)x} \, dx = \frac{\Gamma(n)}{(a+1)^n}, \quad n > 0, \quad a > -1.$$

$$585. \int_0^1 x^m \left(\log \frac{1}{x} \right)^n \, dx = \frac{\Gamma(n+1)}{(m+1)^{n+1}}, \quad m > -1, \quad n > -1.$$

$$586. \int_0^1 x^{m-1} (1-x)^{n-1} \, dx = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} \, dx = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}, \quad n > 0, \quad m > 0.$$

$$587. \int_a^b (x-a)^m (b-x)^n \, dx = (b-a)^{m+n+1} \frac{\Gamma(m+1) \Gamma(n+1)}{\Gamma(m+n+2)},$$

$m > -1, \quad n > -1, \quad b > a.$

$$588. \int_1^\infty \frac{dx}{x^m} = \frac{1}{m-1}, \quad m > 1.$$

$$589. \int_0^\infty \frac{dx}{(1+x)x^p} = \pi \csc p\pi, \quad 0 < p < 1.$$

$$590. \int_0^\infty \frac{dx}{(1-x)x^p} = -\pi \cot p\pi, \quad 0 < p < 1.$$

$$591. \int_0^1 \frac{x^p}{(1-x)^p} \, dx = p\pi \csc p\pi, \quad |p| < 1.$$

$$592. \int_0^1 \frac{x^p}{(1-x)^{p+1}} \, dx = \int_0^1 \frac{(1-x)^p}{x^{p+1}} \, dx = -\pi \operatorname{cosec} p\pi, \quad -1 < p < 0.$$

$$593. \int_0^\infty \frac{x^{p-1}}{1+x} \, dx = \frac{\pi}{\sin p\pi}, \quad 0 < p < 1.$$

594. $\int_0^\infty \frac{x^{m-1}}{1+x^n} dx = \frac{\pi}{n \sin \frac{m\pi}{n}}, \quad 0 < m < n.$

595. $\int_0^\infty \frac{x^a}{(m+x^b)^c} dx = \frac{m^{(a+1-bc)/b}}{b} \frac{\Gamma(\frac{a+1}{b}) \Gamma(c - \frac{a+1}{b})}{\Gamma(c)},$
 $a > -1, b > 0, m > 0, c > \frac{a+1}{b}.$

596. $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}} = \pi.$

597. $\int_0^\infty \frac{a}{a^2+x^2} dx = \begin{cases} \frac{\pi}{2}, & a > 0, \\ \text{or} \\ 0, & a = 0, \\ \text{or} \\ -\frac{\pi}{2}, & a < 0. \end{cases}$

598. $\int_0^a (a^2-x^2)^{n/2} dx = \int_{-a}^a \frac{1}{2} (a^2-x^2)^{n/2} dx = \frac{n!!}{(n+1)!!} \frac{\pi}{2} a^{n+1},$
 $a > 0, n \text{ is an odd integer.}$

599. $\int_0^a x^m (a^2-x^2)^{n/2} dx = \frac{1}{2} a^{m+n+1} \frac{\Gamma(\frac{m+1}{2}) \Gamma(\frac{n+2}{2})}{\Gamma(\frac{m+n+3}{2})}, \quad a > 0, m > -1, n > -2.$

600. $\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \begin{cases} \frac{\sqrt{\pi}}{2} \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n+2}{2})}, & n > -1, \\ \text{or} \\ \frac{(n-1)!!}{n!!} \frac{\pi}{2}, & n \neq 0, n \text{ is an even integer,} \\ \text{or} \\ \frac{(n-1)!!}{n!!}, & n \neq 1, n \text{ is an odd integer.} \end{cases}$

601. $\int_0^\infty \frac{\sin ax}{x} dx = \begin{cases} \frac{\pi}{2}, & a > 0, \\ \text{or} \\ 0, & a = 0, \\ \text{or} \\ -\frac{\pi}{2}, & a < 0. \end{cases}$

602. $\int_0^\infty \frac{\cos x}{x} dx = \infty.$

603. $\int_0^\infty \frac{\tan x}{x} dx = \frac{\pi}{2}.$

604. $\int_0^\infty \frac{\tan ax}{x} dx = \frac{\pi}{2}, \quad a > 0.$

605. $\int_0^\pi \sin(nx) \sin(mx) dx = \int_0^\pi \cos(nx) \cos(mx) dx = 0,$
 $n \neq m, n \text{ is an integer, } m \text{ is an integer.}$

606. $\int_0^{\pi/n} \sin(nx) \cos(nx) dx = \int_0^\pi \sin(nx) \cos(nx) dx = 0, \quad n \text{ is an integer.}$

607. $\int_0^\pi \sin ax \cos bx dx = \begin{cases} \frac{2a}{a^2-b^2}, & a-b \text{ is an odd integer.} \\ \text{or} \\ 0, & a-b \text{ is an even integer.} \end{cases}$

$$\begin{aligned}
608. \int_0^\infty \frac{\sin x \cos ax}{x} dx &= \begin{cases} 0, & |a| > 1, \\ \text{or} \\ \frac{\pi}{4}, & |a| = 1, \\ \text{or} \\ \frac{\pi}{2}, & |a| < 1. \end{cases} \\
609. \int_0^\infty \frac{\sin ax \sin bx}{x^2} dx &= \begin{cases} \frac{\pi a}{2}, & 0 < a \leq b, \\ \text{or} \\ \frac{\pi b}{2}, & 0 < b \leq a. \end{cases} \\
610. \int_0^\pi \sin^2 mx dx &= \int_0^\pi \cos^2 mx dx = \frac{\pi}{2}, \quad m \text{ is an integer.} \\
611. \int_0^\infty \frac{\sin^2 px}{x^2} dx &= \frac{\pi |p|}{2}. \\
612. \int_0^\infty \frac{\sin x}{x^p} dx &= \frac{\pi}{2\Gamma(p) \sin(p\pi/2)}, \quad 0 < p < 1. \\
613. \int_0^\infty \frac{\cos x}{x^p} dx &= \frac{\pi}{2\Gamma(p) \cos(p\pi/2)}, \quad 0 < p < 1. \\
614. \int_0^\infty \frac{1 - \cos px}{x^2} dx &= \frac{\pi |p|}{2}. \\
615. \int_0^\infty \frac{\sin px \cos qx}{x} dx &= \begin{cases} 0, & q > p > 0, \\ \text{or} \\ \frac{\pi}{2}, & p > q > 0, \\ \text{or} \\ \frac{\pi}{4}, & p = q > 0. \end{cases} \\
616. \int_0^\infty \frac{\cos mx}{x^2 + a^2} dx &= \frac{\pi}{2|a|} e^{-|ma|}. \\
617. \int_0^\infty \cos x^2 dx &= \int_0^\infty \sin x^2 dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}. \\
618. \int_0^\infty \sin(ax^n) dx &= \frac{1}{na^{1/n}} \Gamma\left(\frac{1}{n}\right) \sin \frac{\pi}{2n}, \quad n > 1. \\
619. \int_0^\infty \cos(ax^n) dx &= \frac{1}{na^{1/n}} \Gamma\left(\frac{1}{n}\right) \cos \frac{\pi}{2n}, \quad n > 1. \\
620. \int_0^\infty \frac{\sin x}{\sqrt{x}} dx &= \int_0^\infty \frac{\cos x}{\sqrt{x}} dx = \sqrt{\frac{\pi}{2}}. \\
621. \int_0^\infty \frac{\sin^3 x}{x} dx &= \frac{\pi}{4}. \\
622. \int_0^\infty \frac{\sin^3 x}{x^2} dx &= \frac{3}{4} \log 3. \\
623. \int_0^\infty \frac{\sin^3 x}{x^3} dx &= \frac{3\pi}{8}. \\
624. \int_0^\infty \frac{\sin^4 x}{x^4} dx &= \frac{\pi}{3}. \\
625. \int_0^{\pi/2} \frac{dx}{1 + a \cos x} &= \frac{\cos^{-1} a}{\sqrt{1 - a^2}}, \quad |a| < 1.
\end{aligned}$$

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626. $\int_0^\pi \frac{dx}{a + b \cos x} = \frac{\pi}{\sqrt{a^2 - b^2}}, \quad a > b \geq 0.$
627. $\int_0^{2\pi} \frac{dx}{1 + a \cos x} = \frac{2\pi}{\sqrt{1 - a^2}}, \quad |a| < 1.$
628. $\int_0^\infty \frac{\cos ax - \cos bx}{x} dx = \log \left| \frac{b}{a} \right|.$
629. $\int_0^{\pi/2} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{\pi}{2|ab|}.$
630. $\int_0^{\pi/2} \frac{dx}{(a^2 \sin^2 x + b^2 \cos^2 x)^2} = \frac{\pi(a^2 + b^2)}{4a^3b^3}, \quad a > 0, b > 0.$
631. $\int_0^{\pi/2} \sin^{n-1} x \cos^{m-1} x dx = \frac{1}{2} B\left(\frac{n}{2}, \frac{m}{2}\right),$
 m is a positive integer, n is a positive integer.
632. $\int_0^{\pi/2} \sin^{2n+1} x dx = \frac{(2n)!!}{(2n+1)!!}, \quad n$ is a positive integer.
633. $\int_0^{\pi/2} \sin^{2n} x dx = \frac{(2n-1)!!}{(2n)!!} \frac{\pi}{2}, \quad n$ is a positive integer.
634. $\int_0^{\pi/2} \frac{x}{\sin x} dx = 2 \left(\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \dots \right).$
635. $\int_0^{\pi/2} \frac{dx}{1 + \tan^m x} = \frac{\pi}{4}, \quad m$ is a non-negative integer.
636. $\int_0^{\pi/2} \sqrt{\cos x} dx = \frac{(2\pi)^{3/2}}{(\Gamma(1/4))^2}.$
637. $\int_0^{\pi/2} \tan^h x dx = \frac{\pi}{2 \cos(\frac{h\pi}{2})}, \quad 0 < h < 1.$
638. $\int_0^{\pi/2} \frac{\tan^{-1} ax - \tan^{-1} bx}{x} dx = \frac{\pi}{2} \log \frac{a}{b}, \quad a > 0, b > 0.$
639. $\int_0^\infty e^{-ax} dx = \frac{1}{a}, \quad a > 0.$
640. $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \log \frac{b}{a}, \quad a > 0, b > 0.$
641. $\int_0^\infty x^n e^{-ax} dx = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}}, & a > 0, n > -1, \\ \text{or} \\ \frac{n!}{a^{n+1}}, & a > 0, n \text{ is a positive integer.} \end{cases}$
642. $\int_0^\infty x^n e^{-ax^p} dx = \frac{\Gamma((n+1)/p)}{pa^{(n+1)/p}}, \quad a > 0, p > 0, n > -1.$
643. $\int_0^\infty e^{-a^2 x^2} dx = \frac{1}{2a} \sqrt{\pi}, \quad a > 0.$
644. $\int_0^b e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \operatorname{erf}(b\sqrt{a}), \quad a > 0.$
645. $\int_b^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \operatorname{erfc}(b\sqrt{a}), \quad a > 0.$

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646. $\int_0^\infty x e^{-x^2} dx = \frac{1}{2}.$
647. $\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}.$
648. $\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{(2n-1)!!}{2(2a)^n} \sqrt{\frac{\pi}{a}}, \quad a > 0, \quad n > 0.$
649. $\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}, \quad a > 0, \quad n > -1.$
650. $\int_0^1 x^m e^{-ax} dx = \frac{m!}{a^{m+1}} \left[1 - e^{-a} \sum_{r=0}^m \frac{a^r}{r!} \right].$
651. $\int_0^\infty e^{(-x^2 - a^2/x^2)} dx = \frac{e^{-2|a|} \sqrt{\pi}}{2}.$
652. $\int_0^\infty e^{(-ax^2 - b/x^2)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-2\sqrt{ab}}, \quad a > 0, \quad b > 0.$
653. $\int_0^\infty \sqrt{x} e^{-ax} dx = \frac{1}{2a} \sqrt{\frac{\pi}{a}}, \quad a > 0.$
654. $\int_0^\infty \frac{e^{-ax}}{\sqrt{x}} dx = \sqrt{\frac{\pi}{a}}, \quad a > 0.$
655. $\int_0^\infty e^{-ax} \cos mx dx = \frac{a}{a^2 + m^2}, \quad a > 0.$
656. $\int_0^\infty e^{-ax} \cos (bx + c) dx = \frac{a \cos c - b \sin c}{a^2 + b^2}, \quad a > 0.$
657. $\int_0^\infty e^{-ax} \sin mx dx = \frac{m}{a^2 + m^2}, \quad a > 0.$
658. $\int_0^\infty e^{-ax} \sin (bx + c) dx = \frac{b \cos c + a \sin c}{a^2 + b^2}, \quad a > 0.$
659. $\int_0^\infty x e^{-ax} \sin bx dx = \frac{2ab}{(a^2 + b^2)^2}, \quad a > 0.$
660. $\int_0^\infty x e^{-ax} \cos bx dx = \frac{a^2 - b^2}{(a^2 + b^2)^2}, \quad a > 0.$
661. $\int_0^\infty x^n e^{-ax} \sin bx dx = \frac{n! [(a + ib)^{n+1} - (a - ib)^{n+1}]}{2i(a^2 + b^2)^{n+1}}, \quad a > 0.$
662. $\int_0^\infty x^n e^{-ax} \cos bx dx = \frac{n! [(a - ib)^{n+1} + (a + ib)^{n+1}]}{2(a^2 + b^2)^{n+1}}, \quad a > 0, \quad n > -1.$
663. $\int_0^\infty \frac{e^{-ax} \sin x}{x} dx = \cot^{-1} a, \quad a > 0.$
664. $\int_0^\infty e^{-a^2 x^2} \cos bx dx = \frac{\sqrt{\pi}}{2|a|} \exp^{-b^2/(4a^2)}, \quad ab > 0.$
665. $\int_0^\infty e^{-x \cos \phi} x^{b-1} \sin (x \sin \phi) dx = \Gamma(b) \sin (b\phi), \quad b > 0, \quad -\frac{\pi}{2} < \phi < \frac{\pi}{2}.$
666. $\int_0^\infty e^{-x \cos \phi} x^{b-1} \cos (x \sin \phi) dx = \Gamma(b) \cos (b\phi), \quad b > 0, \quad -\frac{\pi}{2} < \phi < \frac{\pi}{2}.$
667. $\int_0^\infty x^{b-1} \cos x dx = \Gamma(b) \cos \left(\frac{b\pi}{2} \right), \quad 0 < b < 1.$

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668. $\int_0^\infty x^{b-1} \sin x \, dx = \Gamma(b) \sin\left(\frac{b\pi}{2}\right), \quad 0 < b < 1.$
669. $\int_0^1 (\log x)^n \, dx = (-1)^n n!, \quad n > -1.$
670. $\int_0^1 \sqrt{\log \frac{1}{x}} \, dx = \frac{\sqrt{\pi}}{2}.$
671. $\int_0^1 \left(\log \frac{1}{x}\right)^n \, dx = n!.$
672. $\int_0^1 x \log(1-x) \, dx = -\frac{3}{4}.$
673. $\int_0^1 x \log(1+x) \, dx = \frac{1}{4}.$
674. $\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n \Gamma(n+1)}{(m+1)^{n+1}}, \quad m > -1, \quad n \text{ is a non-negative integer.}$
675. $\int_0^1 \frac{\log x}{1+x} \, dx = -\frac{\pi^2}{12}.$
676. $\int_0^1 \frac{\log x}{1-x} \, dx = -\frac{\pi^2}{6}.$
677. $\int_0^1 \frac{\log(1+x)}{x} \, dx = \frac{\pi^2}{12}.$
678. $\int_0^1 \frac{\log(1-x)}{x} \, dx = -\frac{\pi^2}{6}.$
679. $\int_0^1 (\log x) \log(1+x) \, dx = 2 - 2 \log 2 - \frac{\pi^2}{12}.$
680. $\int_0^1 (\log x) \log(1-x) \, dx = 2 - \frac{\pi^2}{6}.$
681. $\int_0^1 \frac{\log x}{1-x^2} \, dx = -\frac{\pi^2}{8}.$
682. $\int_0^1 \log\left(\frac{1+x}{1-x}\right) \frac{dx}{x} = \frac{\pi^2}{4}.$
683. $\int_0^1 \frac{\log x}{\sqrt{1-x^2}} \, dx = -\frac{\pi}{2} \log 2.$
684. $\int_0^1 x^m \left[\log\left(\frac{1}{x}\right)\right]^n \, dx = \frac{\Gamma(n+1)}{(m+1)^{n+1}}, \quad m > -1, \quad n > -1.$
685. $\int_0^1 \frac{x^p - x^q}{\log x} \, dx = \log\left(\frac{p+1}{q+1}\right), \quad p > -1, \quad q > -1.$
686. $\int_0^1 \frac{dx}{\sqrt{\log(-\log x)}} = \sqrt{\pi}.$
687. $\int_0^\infty \log\left(\frac{e^x + 1}{e^x - 1}\right) \, dx = \frac{\pi^2}{4}.$
688. $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2.$
689. $\int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \operatorname{cosec} x \, dx = \frac{\pi}{2} \log 2.$

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690. $\int_0^\pi x \log \sin x \, dx = -\frac{\pi^2}{2} \log 2.$
691. $\int_0^{\pi/2} (\sin x) \log \sin x \, dx = \log 2 - 1.$
692. $\int_0^{\pi/2} \log \tan x \, dx = 0.$
693. $\int_0^\pi \log (a \pm b \cos x) \, dx = \pi \log \left(\frac{a + \sqrt{a^2 - b^2}}{2} \right), \quad a \geq b.$
694. $\int_0^\pi \log (a^2 - 2ab \cos x + b^2) \, dx = \begin{cases} 2\pi \log a, & a \geq b > 0, \\ \text{or} \\ 2\pi \log b, & b \geq a > 0. \end{cases}$
695. $\int_0^\infty \frac{\sin ax}{\sinh bx} \, dx = \frac{\pi}{2b} \tanh \frac{a\pi}{2|b|}.$
696. $\int_0^\infty \frac{\cos ax}{\cosh bx} \, dx = \frac{\pi}{2b} \operatorname{sech} \frac{a\pi}{2b}.$
697. $\int_0^\infty \frac{dx}{\cosh ax} = \frac{\pi}{2|a|}.$
698. $\int_0^\infty \frac{x}{\sinh ax} \, dx = \frac{\pi^2}{4a^2}, \quad a \geq 0.$
699. $\int_0^\infty e^{-ax} \cosh (bx) \, dx = \frac{a}{a^2 - b^2}, \quad |b| < a.$
700. $\int_0^\infty e^{-ax} \sinh (bx) \, dx = \frac{b}{a^2 - b^2}, \quad |b| < a.$
701. $\int_0^\infty \frac{\sinh ax}{e^{bx} + 1} \, dx = \frac{\pi}{2b} \csc \frac{a\pi}{b} - \frac{1}{2a}, \quad b \geq 0.$
702. $\int_0^\infty \frac{\sinh ax}{e^{bx} - 1} \, dx = \frac{1}{2a} - \frac{\pi}{2b} \cot \frac{a\pi}{b}, \quad b \geq 0.$
703. $\int_0^{\pi/2} \frac{dx}{\sqrt{1 - k^2 \sin^2 x}} = \frac{\pi}{2} \left[1 + \left(\frac{1}{2} \right)^2 k^2 + \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 k^4 + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \right)^2 k^6 + \dots \right],$
 $k^2 < 1.$
704. $\int_0^{\pi/2} \frac{dx}{(1 - k^2 \sin^2 x)^{3/2}} = \frac{\pi}{2} \left[1 + \left(\frac{1}{2} \right)^2 3k^2 + \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 5k^4 \right.$
 $\left. + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \right)^2 7k^6 + \dots \right], \quad k^2 < 1.$
705. $\int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 x} \, dx = \frac{\pi}{2} \left[1 - \left(\frac{1}{2} \right)^2 k^2 - \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 \frac{k^4}{3} \right.$
 $\left. - \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \right)^2 \frac{k^6}{5} - \dots \right], \quad k^2 < 1.$
706. $\int_0^\infty e^{-x} \log x \, dx = -\gamma.$
707. $\int_0^\infty e^{-x^2} \log x \, dx = -\frac{\sqrt{\pi}}{4} (\gamma + 2 \log 2).$
708. $\int_0^\infty \left(\frac{1}{1 - e^{-x}} - \frac{1}{x} \right) e^{-x} \, dx = \gamma.$

709. $\int_0^\infty \frac{1}{x} \left(\frac{1}{1-e^{-x}} - \frac{1}{x} \right) dx = \gamma.$

5.5.1 TABLE OF SEMI-INTEGRALS

	f	$\frac{d^{-1/2} f}{dx^{-1/2}}$
(1)	0	0
(2)	1	$2\sqrt{\frac{x}{\pi}}$
(3)	$x^{-1/2}$	$\sqrt{\pi}$
(4)	x	$\frac{4x^{2/3}}{3\sqrt{\pi}}$
(5)	$x^n, n = 0, 1, 2, \dots$	$\frac{(n!)^2 (4x)^{n+1/2}}{(2n+1)! \sqrt{\pi}}$
(6)	$x^p, p > -1$	$\frac{\Gamma(p+1)}{\Gamma(p+\frac{3}{2})} x^{p+1/2}$
(7)	$\sqrt{1+x}$	$\sqrt{\frac{x}{\pi}} + \frac{(1+x) \tan^{-1}(\sqrt{x})}{\sqrt{\pi}}$
(8)	$\frac{1}{\sqrt{1+x}}$	$\frac{2}{\sqrt{\pi}} \tan^{-1}(\sqrt{x})$
(9)	$\frac{1}{1+x}$	$\frac{2 \sinh^{-1}(\sqrt{x})}{\sqrt{\pi(1+x)}}$
(10)	e^x	$e^x \operatorname{erf}(\sqrt{x})$
(11)	$e^x \operatorname{erf}(\sqrt{x})$	$e^x - 1$
(12)	$\sin(\sqrt{x})$	$\sqrt{\pi x} J_1(\sqrt{x})$
(13)	$\cos(\sqrt{x})$	$\sqrt{\pi x} H_{-1}(\sqrt{x})$
(14)	$\sinh(\sqrt{x})$	$\sqrt{\pi x} I_1(\sqrt{x})$
(15)	$\cosh(\sqrt{x})$	$\sqrt{\pi x} L_{-1}(\sqrt{x})$
(16)	$\frac{\sin(\sqrt{x})}{\sqrt{x}}$	$\sqrt{\pi} H_0(\sqrt{x})$
(17)	$\frac{\cos(\sqrt{x})}{\sqrt{x}}$	$\sqrt{\pi} J_0(\sqrt{x})$
(18)	$\log x$	$2\sqrt{\frac{x}{\pi}} [\log(4x) - 2]$
(19)	$\frac{\log x}{\sqrt{x}}$	$\sqrt{\pi} \log\left(\frac{x}{4}\right)$