

1. The most important method of energy production in the sun is the fusion of protons into  $4\text{He}$  nuclei. This predominantly takes place through the reactions  $p + p \rightarrow d + e^+ + \nu_e$ ,  $p + d \rightarrow {}^3\text{He} + \gamma$  and  ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + p + p$ . A total of  $28.3\text{ MeV}$  is released for every  ${}^4\text{He}$  nucleus produced. 90% of this energy exits as electromagnetic radiation and the rest is mostly converted into the kinetic energy of neutrinos (typically  $0.4\text{ MeV}$ ).
  - (a) What is the flux of solar neutrinos at the earth (distance from the sun:  $a_0 = 1.5 \times 10^8\text{ km}$ )?
  - (b) In a tunnel in the Abruzzi the GALLEX experiment measures neutrinos through the reaction  ${}^{71}_{31}\text{Ga} + \nu_e \rightarrow {}^{71}_{32}\text{Ge} + e^-$ . The cross-section of this reaction is about  $2.5 \times 10^{-45}\text{ cm}^2$ . One looks for radioactive  ${}^{71}\text{Ge}$  atoms (lifetime  $\tau = 16\text{ days}$ ) which are produced in a tank containing  $30\text{ t}$  of dissolved gallium (40%  ${}^{71}\text{Ga}$ , 60%  ${}^{69}\text{Ga}$ ) chloride. About 50% of the neutrinos have an energy above the reaction threshold. One extracts all the germanium atoms from the tank. Estimate how many  ${}^{71}\text{Ge}$  atoms are produced each day and after three weeks? How many if one waits forever?
  
2. A neutron star with mass,  $M = 1.5M_{\text{sol}}$  ( $\approx 3.0 \times 10^{30}\text{ kg}$ ), and radius  $R \approx 10\text{ km}$  is the remnant of a supernova. The stellar material originates from the iron core ( $R \gg 10\text{ km}$ ) of the supernova.
  - (a) How much energy was released during the lifetime of the original star by converting hydrogen into iron? (The binding energy of  ${}^{56}\text{Fe}$  is  $B = 8.79\text{ MeV/nucleon}$ .) NB: Since after the implosion only a part of the original iron core remains in the neutron star, the calculation should be performed only for this mass.
  - (b) How much energy was released during the implosion of the iron core into a neutron star?
  - (c) In what form was the energy radiated off?